The second Japanese-Australian Workshop on Real and Complex Singularities

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Abstracts

Carles Bivià-Ausina (València)
Mixed multiplicities of ideals and the effective computation of Łojasiewicz exponents of complex analytic maps

Abstract: Let $g : (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ be a finite analytic map. Let us denote by $m_0(g)$ the multiplicity of g. Under a condition on the numerical value of $m_0(g)$, we give an expression for the Łojasiewicz exponent $\mathcal{L}_0(g)$ in terms of the σ-multiplicities of the powers of certain monomial ideals attached to g. The σ-multiplicity of n ideals, not necessarily of finite colength, in a local ring of dimension n is a notion that we have defined and that extends the notion of mixed multiplicity defined by Rees. Therefore the notions of joint reduction of ideals introduced by Rees in the 80's and its relation with the integral closure of ideals play a fundamental role in our work.

If the multiplicity $m_0(g)$ is known, then the expression of $\mathcal{L}_0(g)$ that we give leads to an effective method to compute $\mathcal{L}_0(g)$. This method is based on the computation of the multiplicity of monomial ideals and the Rees formula relating mixed multiplicities with Samuel multiplicities.

We will apply our results to the computation of Łojasiewicz exponents of gradient maps in some examples. We remark that the above mentioned condition on the numerical value of $m_0(g)$ lead us to the definition of a class of functions $(\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ that includes semi-weighted-homogeneous and Newton non-degenerate functions, in the sense of Kouchnirenko. We will also show a characterization of the functions of this ampler class in terms of the Newton polyhedra of its partial derivatives.

Alex Dimca (Nice)
Characteristic and resonance varieties

Abstract: The characteristic varieties $V_k(M)$ describe the jumping loci of the coho-

mology of a complex smooth algebraic variety M with coefficients in rank one local systems.

The associated resonance variety $R_k(M)$ is a linear approximation of the characteristic variety $V_k(M)$, e.g. in many cases it is just the tangent cone of the singularity $(V_k(M), 1)$, where 1 denotes the trivial character.

In a joint paper with S. Papadima and A. Suciu , we have studied the properties of this class of singularities, and their implications on the fundamental group $\pi_1(M)$.

Michael Eastwood (Adelaide)
Singularities of semiconformal mappings

Abstract: A submersion between two Riemannian manifolds is said to be semiconformal if it is conformal orthogonal to the fibres. Of particular interest are semiconformal mappings from Euclidean 3-space to Euclidean 2-space where interesting singularities and critical points can develop. I shall discuss joint work with Paul Baird, which produces many examples of such mappings and the various conformal invariants that control them. Some of these invariants have separate geometric significance.

Alexander Esterov (Moscow)
Poincare-Hopf indices and Newton polyhedra

Abstract: Oka's formula describes the Poincare-Hopf index of a complex analytic vector field in terms of the Newton polyhedra of its components. In this talk, we discuss the geometric meaning of the right hand side of this formula, extend this formula to some generalized versions of the Poincare-Hopf index, and present its counterpart for real vector fields.

Adam Harris (Armidale)
Higher analytic cohomology and the Laplace-Beltrami operator near an isolated singularity

Abstract: Let X be a complex analytic subvariety of dimension n in $\mathbb{C}^N$, with isolated singularity at the origin. Moreover let us assume that the Kahler metric on the smooth locus of X induced by that of the ambient space has L^n-curvature in a neighbourhood of the origin. In earlier joint work with Y. Tonegawa it was shown that a smooth $\bar{\partial}$-closed differential form of type $(0, 1)$ is exact if it is also L^n in a neighbourhood of the singularity. This result is based on work of S. Bando, who extended Kohn's theory of the $\bar{\partial}$-Neumann problem on strongly pseudoconvex domains to the punctured ball in $\mathbb{C}^2$. Our straightforward generalisation of Bando's ideas was applied to domains

with one strongly pseudoconvex and one strongly pseudoconcave component. In the present talk we further develop this work to address higher Dolbeault cohomology, and examine its relationship to harmonic forms in the same context.

Goo Ishikawa (Sapporo)
Local classification of varieties in the symplectic space

Abstract: A survey on the classification of varieties in the symplectic space will be given. In particular, the classification of Whitney umbrellas in the symplectic four spaces will be discussed as a prototype of the study. The relation to the classification of glancing hypersurfaces by Melrose will be mentioned. Also the results on Lagrangian varieties will be reviewed along the general theory of the symplectic classification.

Shyuichi Izumiya (Sapporo)
Flat lightlike hypersurcface in Minkowski space

Abstract: The study of the extrinsic differential geometry of submanifolds in Minkowski space is of special interest in Relativity Theory. In particular, lightlike hypersurfaces, which can be constructed as ruled hypersurfaces whose induced first fundamenal forms are positive semi-defnite, provide good models for the study of different horizon types ([1], [7]). It is also given as a ruled hypersurface along a spacelike submanifold of codimension two whose rulings are lightlike lines. In the previous paper [3], we provided a linkage to the differential geometry of lightlike hypersurfaces in Minkowski 4-space with the modern theory of Legendrian singularities and classified the singularities in generic. As a consequence, the list of singularities for generic lightlike hypersurfaces is the same as that of wave fronts. We have also studied the Lorentzian geometric meaning of such singularities. Inspired by the results of M. Kossowski ([5, 6]), we defined the notion of lightcone Gauss map on its associated spacelike surface. By using the lightcone Gauss map, we have also defined the lightcone Gauss-Kronecker curvature of the spacelike surface [2, 4]. This curvature can be considered as a curvature of the lightlike hypersurface itself which is a conformal invariant. However the flatness for this curvature is a Lorentzian invariant property. Therefore, the notion of flat lightlike hypersurface is a welldeifned notion of a Lorentzian geometry. Here we say that a lightlike hypersurface is flat if the lightcone Gauss-Kronecker curature vanishes at any point on the sapcelike surface. In this talk, we consider a flat lightlike hypersurface. The singularities of such hypersurfaces are different from the general lightlike hypersurfaces. There appeared $D_4^{\pm}$-type singularities for general lightlike hypersurfaces. However, flat lightlike hypersurfaces have only A_k-type singularities in generic. We construct a general framework for the study of flat lightlike hypersurfaces and give a classification of singularities in generic. This is a joint work with Carmen Romero-Fuster and Kentaro Saji.

References

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Boldizsár Kalmár (Fukuoka)
Fold maps, immersions and cobordism

Abstract: We study global topological properties of singular maps with only fold singularities between smooth manifolds. We define geometrical cobordism invariants of these maps in terms of immersions with prescribed normal bundles, study the cobordism group of fold maps considering local and global restrictions on the set of allowed singular fibers. We compute several cobordism groups and show relations between the topological properties of the source and target manifolds and the topology of the singular set (and the set of singular fibers), mainly from the viewpoint of cobordism.

Tzee-Char Kuo (Sydney)
Arc Space, Morse Stability, Equi-singularities (Joint work with L. Paunescu)

Abstract: The Newton-Puiseux convergent fractional power series is used as coordinates for the space of arcs. An extension of the Morse Stability Theorem is established in this space, leading to the notion of Morse equi-singular deformation.

For example, the family $x^4-t^2x^2y^2+y^2$, which is stable in the usual sense (μ-constant), is *not* Morse stable. This corresponds to the fact that $p_t(x) := x^4 - t^2x^2 + 1$ in not Morse stable in the usual sense. Here $x = 0$ splits into three critical points as t varies away from 0; however, the zero set (variety) is stable, consisting always of four distinct roots (two in $\mathbb{R}$).

Takashi Nishimura (Yokohama)
An upper bound for multiplicities of co-rank one simple singularities

Abstract: We present an upper bound for the multiplicity $\delta(f)$ of a co-rank one $\mathcal{A}$-simple singularity $f : (\mathbf{R}^n, 0) \to (\mathbf{R}^p, 0)$ $(n \leq p)$. Our upper bound depends only on dimensions of the source and the target spaces, and it gives in fact the maximal value of multiplicities of co-rank one $\mathcal{A}$-simple singularities in all known classification results except the cases that $n = 1, p \geq 6$. On the other hand, for any n if $p \geq 7n$ then the maximal value of multiplicities of co-rank one $\mathcal{A}$-simple singularities fails to attain our upper bound.

Mutsuo Oka (Tokyo)
Topology of linear system of plane curves

Abstract: We study the geometry of pencil of plane curves span by two smooth curves C, C' of degree d. The location of the base locus $C \cap C'$ plays an important role. Two particular cases are studied: the generic case where the base locus consists of d^2 points and another extreme case where the base locus is a single point. In the second case, we will show that the Taylor expansion plays a key role.

Adam Parusiński (Angers)
Equivalence relations of two variable real analytic function germs

Abstract: We give complete characterisations of blow-analytic equivalence for two variable real analytic function germs in terms of their minimal resolution and their real tree models. We compare also this equivalence to other natural equivalence relations, particularly to the bi-lipschitz and C^1 ones.

Laurentiu Paunescu (Sydney)
Tame nonsmooth inverse mapping theorems

Abstract: We show several versions of the inverse mapping theorem for tame nonsmooth mappings. Here a tame mapping means a mapping which is subanalytic or definable in some o-minimal structure.

Osamu Saeki (Fukuoka)
Cobordism of Morse maps and its application to map germs

Abstract: This is a joint work with Kazuichi Ikegami (Okayama Univ. of Science).

Let $f : M \to S^1$ be a Morse map of a closed manifold M into the circle, where a Morse map is a smooth map with only nondegenerate critical points. In this talk, we classify such maps up to fold cobordisms. In the course of the classification, we get several fold cobordism invariants for such Morse maps. We also consider a slightly general situation where the source manifold M has boundary and the map f restricted to the boundary has no critical points.

Let us consider a generic smooth map germ $g : (\mathbf{R}^m, 0) \to (\mathbf{R}^2, 0)$ with $m \geq 2$, where the target $\mathbf{R}^2$ is oriented. Then, using the above-mentioned fold cobordism invariants, we show that the number of cusps with prescribed indices appearing in a stable perturbation of g, counted with sign, is a local topological invariant of g.

Kentaro Saji (Sapporo)
Singularities of de Sitter horocyclic surfaces

Abstract: One-parameter families of circles are called *circular surfaces*. Circular surfaces appear in some geometrical context, and recently, they are investigated from the view point of the singularity theory ([1, 2]). In this talk, we study the one-parameter families of "de Sitter horocycles" in de Sitter 3-space. Let $(\boldsymbol{R}^4_1, \langle\ ,\ \rangle)$ be the Lorentz-Minkowski 4-space with signature $(-,+,+,+)$. Let $\boldsymbol{a}_0(t), \ldots, \boldsymbol{a}_3(t)$ be a pseudo-orthonormal frame field in $\boldsymbol{R}^4_1$ such that $\langle \boldsymbol{a}_0, \boldsymbol{a}_0 \rangle \equiv -1$ and $\langle \boldsymbol{a}_1, \boldsymbol{a}_1 \rangle \equiv \langle \boldsymbol{a}_2, \boldsymbol{a}_2 \rangle \equiv \langle \boldsymbol{a}_3, \boldsymbol{a}_3 \rangle \equiv 1$. We assume that $\langle \boldsymbol{a}_2', \boldsymbol{a}_2' \rangle \neq 0$ and the parameter t is arc-length with respect to the curve $\boldsymbol{a}_2$.

Consider a surface

$$X(s,t) = \boldsymbol{a}_2(t) + s\boldsymbol{a}_1(t) - (s^2/2)\boldsymbol{l}(t),$$

where, $\boldsymbol{l} = \boldsymbol{a}_0 + \boldsymbol{a}_2$. Then X can be regarded as a one-parameter family of de Sitter horocycles. Furthermore, since

$$\langle X, X \rangle = \big\langle \boldsymbol{a}_2 + s\boldsymbol{a}_1 - s^2(\boldsymbol{a}_0 + \boldsymbol{a}_2)/2, \boldsymbol{a}_2 + s\boldsymbol{a}_1 - s^2(\boldsymbol{a}_0 + \boldsymbol{a}_2)/2 \big\rangle = 1$$

holds, it holds that $X : \boldsymbol{R}^2 \longrightarrow S^3_1 = \{x \in \boldsymbol{R}^4_1 | \, \langle x, x \rangle = 1\}$. Here, S^3_1 is called the *de Sitter 3-space*. We study singularities and properties of the surface X.

We put $c_1(t) = \langle \boldsymbol{a}_0', \boldsymbol{a}_1 \rangle$, $c_2(t) = \langle \boldsymbol{a}_0', \boldsymbol{a}_2 \rangle$, $c_3(t) = \langle \boldsymbol{a}_0', \boldsymbol{a}_3 \rangle$, $c_4(t) = \langle \boldsymbol{a}_1', \boldsymbol{a}_2 \rangle$, $c_5(t) = \langle \boldsymbol{a}_1', \boldsymbol{a}_3 \rangle$ and $c_6(t) = \langle \boldsymbol{a}_2', \boldsymbol{a}_3 \rangle$. Then invariant functions c_* satisfy the following Frenet-

Serre type formulae:

$$\begin{pmatrix} \boldsymbol{a}_0' \\ \boldsymbol{a}_1' \\ \boldsymbol{a}_2' \\ \boldsymbol{a}_3' \end{pmatrix} = \begin{pmatrix} 0 & c_1 & c_2 & c_3 \\ c_1 & 0 & c_4 & c_5 \\ c_2 & -c_4 & 0 & c_6 \\ c_3 & -c_5 & -c_6 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_0 \\ \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \end{pmatrix}.$$

Using these invariants and formulae, we study the properties and singularities of X.

References

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Vyacheslav Sedykh (Moscow)

The coexistence of real corank 1 singularities and the contact geometry of space curves

Abstract: Let M and N be smooth closed manifolds of dimensions m and n, respectively, where $n \geq m$. Consider stable smooth mappings $M \to N$ having only singularities of corank 1. We find a complete system of universal linear relations between the Euler characteristics of the manifolds of multi-singularities of mappings under consideration. As an application, we obtain multidimensional generalizations of the Bose theorem on supporting circles of a plane curve and multidimensional generalizations of the Freedman-Banchoff-Gaffney-McCrory formula for the number of triple tangent planes of a curve in $\mathbb{R}^3$.

Jim Ward (Sydney)

Lines on a cubic surface

Abstract: I present a talk on straight lines on a cubic surface. The existence of these and their configulation will be demonstrated by considerring degenerate plane cubic curve which occur as the intersection of a plane with a generic cubic surface.

Minoru Yamamoto (Kurume)

On pseudo-immersions of a surface into the plane

Abstract: Let M be a compact oriented surface with exactly one boundary component. A smooth map of M into the plane is said to be a pseudo-immersion if it has only fold singularities and is an orientation-preserving immersion near the boundary of M. In

this talk, for a pseudo-immersion we study the relation between the normal degree of the boundary and the number of the singularity components.

Ruibin Zhang (Sydney)

Representations of Quantum Groups and Cohomology of Quantum Equivariant Vector Bundles

Abstract. Quantum group equivariant vector bundles on quantum homogeneous spaces are classified. An analogue of Dolbeault cohomology is presented for such noncommutative vector bundles. A quantum version of the Bott-Borel-Weil theorem is discussed within this noncommutative geometric framework, realizing the irreducible representations of quantized universal enveloping algebras on the cohomology groups.