The sixth Japanese-Australian Workshop on Real and Complex Singularities

Date: 23 — 27 November, 2015
Place: The main meeting room of Faculty of Science, Kagoshima University
Organisers: Laurentiu Paunescu, Adam Harris, Alex Isaev, Satoshi Koike, Kimio Miyajima, Shoji Yokura, Toshizumi Fukui

Program

23 November, Monday

14:50 Opening

15:00 — 16:00 Toru Ohmoto (Hokkaido University)
C^1-triangulations of semialgebraic sets and de Rham homotopy theory

16:10 — 17:10 Cristina Valle (Tokyo Metropolitan University)
On the blow-analytic classification of surface singularities

24 November, Tuesday

9:40 — 10:40 Michael Barwick (University of Sydney)
Morse classification of low order jet spaces

10:50 — 11:50 Mike Eastwood (University of Adelaide)
Isolated hypersurface singularities, associated forms, and their invariants

13:40 — 14:40 Shinichi Tajima (Tuskuba University)
Limiting tangent spaces, Teissier sequences μ^* and parametric local cohomology.

14:50 — 15:50 Jiro Sekiguchi (Tokyo University of Agriculture and Technology)
Deformations of a plane curve related with the Valentiner group

16:00 — 17:00 Yuichi Ike (University of Tokyo)
Lefschetz fixed point formulas for higher-dimensional fixed point sets

25 November, Wednesday

9:40 — 10:40 Keisuke Teramoto (Kobe University)
Principal curvatures and generic behavior of wave fronts

10:50 — 11:50 Pei Donghe (Northeast Normal University)
The singularities of spacelike surfaces in Anti de Sitter space

Afternoon (Free discussion)

26 November, Thursday

9:40 — 10:40 Kenta Hayano (Hokkaido University)
Lefschetz fibrations and mapping class groups

10:50 — 11:50 Kazuto Takao (Kyushu University)
Local moves of the Stein factorization of the product map of two functions

13:40 — 14:40 Jonathan Hillman (University of Sydney)
Complements of connected hypersurfaces in S^4

14:50 — 15:50 Mutsuo Oka (Tokyo University of Science)
On the roots of an extended Lens equation and an application

16:00 — 17:00 Alex Dimca (Université de Nice-Sophia Antipolis)
On free and nearly free plane curves

19:00 – (Conference dinner at Wakana)

27 November, Friday

9:40 — 10:40 Rui Bin Zhang (University of Sydney)
Invariant theory of the orthosymplectic and special orthosymplectic supergroups

10:50 — 11:50 Shuzo Izumi
Sufficiency of simplex inequalities

13:40 — 14:40 Adam Parusiński (Université de Nice-Sophia Antipolis)
Proof of Whitney's fibering conjecture

14:50 — 15:50 Laurentiu Paunescu (University of Sydney)
SSP sets and their bilipschitz properties

16:00 — 17:00 Krzysztof Kurdyka (Université de Savoie)
Curve-rational functions

本研究集会は次の研究費の支援を受けています。
科研費基盤研究 A 写像の特異点論の新展開（研究代表者 佐伯修）課題番号：23244008
科研費基盤研究 B 代数的特異点の大域的研究（研究代表者 小池敏司）課題番号：26287011
科研費基盤研究 C 特異点論から見た曲面論と派生する微分方程式の研究（研究代表者 福井敏純）課題番号：15K04867

Abstracts of talks in JARCS6

Michael Barwick (University of Sydney)
Morse classification of low order jet spaces

Alex Dimca (Université de Nice-Sophia Antipolis)
On free and nearly free plane curves

We discuss two geometric features of free and nearly free curves: the relation with rational cuspidal curves and the maximality of their Tjurina numbers.

Mike Eastwood (University of Adelade)
Isolated hypersurface singularities, associated forms, and their invariants

To a complex isolated hypersurface singularity one can associate a finite-dimensional commutative algebra and, in case this algebra has suitable properties inherited from the nature of the singularity, construct "associated forms" for this algebra, and then invariants of these forms, which, by the nature of this route, will be invariants of the original singularity. I shall discuss, mainly by means of examples, the work of Alexander Isaev and his co-workers in carving out this route.

Kenta Hayano (Hokkaido University)
Lefschetz fibrations and mapping class groups

Lefschetz fibrations are smooth maps on 4-manifolds which were originally studied in complex/algebraic geometry. However, after seminal works of Donaldson and Gompf relating Lefschetz fibrations to symplectic structures, these fibrations have attracted a lot of interest from 4-dimensional topologists. In this talk, we first explain relation between Lefschetz fibrations and mapping class groups (i.e. groups of isotopy classes of self-diffeomorphisms) of surfaces. We then apply this relation to several problems on Lefschetz fibrations and symplectic 4-manifolds.

Jonathan A. Hillman (University of Sydney)
Complements of connected hypersurfaces in S^4

We consider the Euler characteristics and fundamental groups of the complementary regions X and Y of an embedding of a closed 3-manifold M in $S^4 = X \cup_M Y$. Our main tools are the Massey structure in $H^*(M;\mathbb{Z})$ and a theorem of Stallings on the lower central series. The main examples derive from 0-framed link presentations for M. We also use TOP surgery to propose a characterization of the simplest embeddings of $F \times S^1$, where F is a closed orientable surface.

Yuichi Ike (University of Tokyo)
Lefschetz fixed point formulas for higher-dimensional fixed point sets

We study the Lefschetz fixed point formula for constructible sheaves with higher-dimensional fixed point sets. Under certain assumptions, we prove that the local contributions from them are expressed by Euler integrals of explicitly computable constructible functions. In the course of the proof, the theory of Lagrangian cycles is effectively used. Our result can be applied to the Lefschetz fixed point formula for singular spaces. This is a joint work with Yutaka Matsui and Kiyoshi Takeuchi.

Shuzo Izumi
Sufficiency of simplex inequalities

Let $z_0, \dots, z_n$ be the $(n-1)$-dimensional volumes of facets of an n-simplex. Then we have the simplex inequalities: $z_p < z_0 + \cdots + \check{z}_p + \cdots + z_n$ $(0 \leq p \leq n)$, generalizations of the triangle inequalities. Conversely, suppose that numbers $z_0, \dots, z_n > 0$ satisfy these inequalities. Does there exist an n-simplex the volumes of whose facets are them? Kakeya solved this problem affirmatively in the case $n = 3$ and conjectured the assertion for all $n \geq 4$. We prove that his conjecture is affirmative.

Krzysztof Kurdyka (Université de Savoie)
Curve-rational functions

Let X be an algebraic subset of $\mathbb{R}^n$ and $f : X \to \mathbb{R}$ a semialgebraic function. We prove that if f is continuous rational on each curve $C \subset X$ then: 1) f is arc-analytic, 2) f is continuous rational on X. As a consequence we obtain a characterization of hereditarily rational functions recently studied by J. Kollár and J.K. Nowak,. The last paper is related to a recent work of C. Fefferman and J. Kollar on solutions of linear systems with polynomial coefficients. Joint work with W. Kucharz

Toru Ohmoto (Hokkaido University)
C^1-triangulations of semialgebraic sets and de Rham homotopy theory

This is a joint work with Masahiro Shiota. We show that any semialgebraic set admits a semialgebraic triangulation such that each closed simplex is C^1 differentiable. As an application, we give a straightforward definition of the integration $\int_X \omega$ over a compact semialgebraic subset X of a differential form ω on an ambient semialgebraic manifold, that provides a significant simplification of the theory of semialgebraic singular chains and integrations without using geometric measure theory. For instance, we can easily access to the semialgebraic de Rham homotopy theory outlined by Kontsevich-Soibelman. Our results hold over any (possibly non-archimedian) real closed field.

Mutsuo Oka (Tokyo University of Science)
On the roots of an extended Lens equation and an application

We consider a certain mixed polynomial which is an extended Lens equation $L_{n,m} = \bar{z}^m - p(z)/q(z)$ with deg $q(z) = n$, deg $p(z) < n$ whose numerator is a mixed polynomial of degree $(n + m; n, m)$. Then we consider its deformation of the type $L_{n,m} + \varepsilon/z^m$ to construct a special mixed polynomial of degree $(n + 2m; n + m, m)$ with $5n$ zeros. This generalizes an example of Rhie. We give an application to the number of moduli spaces of given strongly mixed weighted homogeneous polynomials of two variables.

Adam Parusiński (Université de Nice-Sophia Antipolis)
Proof of Whitney's fibering conjecture

Varchenko showed that if a real or complex analytic set is Zariski equisingular along an affine subspace T then it is locally topologically trivial along T. Using Whitney interpolation we construct a trivialization of a Zariski equisingular set that is moreover analytic on real analytic arcs and analytic, real resp. complex, with respect to the parameter space T.

Then given an algebraic set, or a germ of an analytic set, we construct its stratification that locally along each stratum fibers the set into, real resp. complex, analytic submanifolds with the strong continuity of tangent spaces, analogous to Verdier's condition (w). This shows Whitney's fibering conjecture (only local version in the analytic case). Our construction is algorithmic, involves only linear changes of coordinates and computation of subsequent discriminants. This is a joint work with Laurentiu Paunescu.

Laurentiu Paunescu (University of Sydney)
SSP sets and their bilipschitz properties.

In this talk I will define the SSP sets and show that they behave well under biLipschitz homeomorphisms. In particular we will show that two SSP sets who are bilipschitz equivalent have their tangent cones bilipschitz equivalent as well. (This is joint work with Satoshi Koike)

Donghe Pei (Northeast Normal University)
The singularities of spacelike surfaces in Anti de Sitter space

In this talk, we study the singularities of spacelike surfaces in Anti de Sitter space by the lightlike Gauss-Kronecker curvature. And we find the equivalent conditions of inflection points of real type, imaginary type or flat type from the view point of lightlike geometry.

Jiro Sekiguchi (Tokyo University of Agriculture and Technology)
Deformations of a plane curve related with the Valentiner group

H. Valentiner studied in detail a group of order 360 generated by homologies. Wiman showed that the group studied by Valentiner is isomorphic to the algernating group of 6-th degree. This group leads to a complex reflection group of order 2160 generated by 45 2-fold reflections. We denote by G_{27} the complex reflection group since it apperas in the list of the paper by Shephard-Todd as the 27-th group. It is known that the rank of G_{27} is 3 and that the degrees of basic G_{27}-invariants are 6,12,30.

In this talk, we discuss the problem to construct a family of hypersurfaces related with the group G_{27}. Before stating the result, we explain the meaning of the problem. We recall the case of simple singularities. There are many studies on the relationship between families of deformations of simple hypersurface singularities and Weyl groups or root systems of types A_n, D_n, E_n. Then the problem is this: take G_{27} instead of the Weyl group. In this case, what should be taken as the corresponding family of hypersurfaces? My interpretation is as follows.

Introduce a family of plane curves in (u,v)-plane defined by

$$C(x_1,\ldots,x_5,y) : u^5 + u^2v^3 + x_1u^4 + x_2u^3 + x_3u^2 + x_4u + x_5 + u^2v^3 - 3yuv^2 + 3y^2v = 0.$$

Note that $(x_1, \ldots, x_5, y)$ is a parameter. Return to the group G_{27}. There basic G_{27}-invariant polynomials I_6, I_{12}, I_{30} of degree 6,12,30, respectively. Then there is a polynomial $F(z_1, z_2, z_5)$ of z_1, z_2, z_5 such that $F(I_6, I_{12}, I_{30})$ is the discriminant of G_{27}. We define weighted homogeneous polynomials of z_1, z_2, z_5 by

$$\begin{aligned} X_1 &= a_1 z_1, \\ X_2 &= a_2 z_2 + b_1 z_1^2, \\ X_3 &= a_3 z_1 z_2 + a_4 z_1^3, \\ X_4 &= a_5 z_2^2 + a_6 z_1^2 z_2 + a_7 z_1^4, \\ X_5 &= a_8 z_5 + b_2 z_1 z_2^2 + b_3 z_1^3 z_2 + b_4 z_1^5, \\ Y &= a_9 z_2 + a_{10} z_1^2, \end{aligned}$$

where $a_1, a_2, \cdots, b_1, b_2, \cdots$ are constants and $a_1 a_2 a_8 \neq 0$. Then by a suitable choice of the constants $a_1, a_2, \cdots, b_1, b_2, \cdots$, we find that the singular locus of the family of curves $C(X_1, \cdots, X_5, Y)$ with parameter (z_1, z_2, z_5) coincides with the hypersurface defined by $F(z_1, z_2, z_5) = 0$. This means that the family of curves $C(X_1, \cdots, X_5, Y)$ is regarded as deformations of the curve $u^5 - u^2 v^3 = 0$ corresponding to the group G_{27}.

S. Tajima and K. Nabeshima (Tuskuba Univesity and Tokushima University)
Limiting tangent spaces, Teissier sequences μ^* and parametric local cohomology

Conormal geometry of limiting tangent spaces of hypersurface isolated singularities are considered. A new method for computing limiting tangent spaces is presented. The resulting algorithm provides stratifications of limiting tangent spaces according to the relevant multiplicities. A key of the method is the use of the concept of parametric local cohomology systems. An application to the computation of the Teissier sequence is also given.

Kazuto Takao (Kyushu University)

Local moves of the Stein factorization of the product map of two functions

The Stein factorization is useful to describe the topological behavior of a map. That of a stable map from a 3-manifold to the plane is a 2-dimensional cell-complex. In this talk, I give some local moves of it which can be realized by a homotopy of the map. Moreover, they can be realized preserving the product structure. That is to say, I give some local moves of the Stein factorization of the product map of two functions on a 3-manifold which can be realized by isotopies of the functions. I also talk about the relation of this work to Heegaard theory of 3-manifolds.

Keisuke Teramoto (Kobe University)
Principal curvatures and generic behavior of wave fronts

In this talk, I deal with the behavior of principal curvatures of wave fronts near non-degenerate singular points which include cuspidal edges and swallowtails. Several geometric invariants and the definition of generic behavior of wave fronts have been obtained in previous works. I also talk about relationships among behavior of principal curvatures and generic behavior of wave fronts near non-degenerate singular points.

Cristina Valle (Tokyo Metropolitan University)
On the blow-analytic classification of surface singularities

We investigate the blow-analytic equivalence of real isolated surface singularities. By extending the definition of the topological invariant μ originally introduced for the study of curve singularities embedded in a smooth surface, we are able to give a first classification of surface singularities up to blow-analytic homeomorphism. In this talk, we describe our method and latest results, and outline a possible refinement of the invariant. We also produce explicit examples of surface singularities corresponding to $\mu = 1$ and $\mu = 2$.

Rui Bin Zhang (university of Sydney)
Invariant theory of the orthosymplectic and special orthosymplectic supergroups

The first and second fundamental theorems (FFT and SFT) of invariant theory for the orthosymplectic supergroup $\mathrm{OSp}(V)$ can be most conveniently formulated in terms of the Brauer category $\mathcal{B}(d)$ (where the parameter d is equal to the super-dimension of V). There exists a full tensor functor from $\mathcal{B}(d)$ to the category of finite dimensional $\mathrm{OSp}(V)$-representations, and the kernel of the restriction of th e functor to morphisms can also be described explicitly. However, the invariant theory for the special orthosymplectic supergroup $S\mathrm{OSp}(V)$ is more intricate. We will introduce some $S\mathrm{OSp}(V)$-invariants, which are referred to as super Pfaffians, and show that they together with the Brauer-like invariants generate all the $S\mathrm{OSp}(V)$-invariants. We present the results by introducing a tensor category containing $\mathcal{B}(d)$ and constructing a full tensor functor from it to the category of finite dimensional $S\mathrm{OSp}(V)$-representations. This talk is based on joint papers with Lehrer and Deligne.