

# Hyper JARCS

Memorial conference for Professor Stefan Papadima

2-6 December, 2019

Room 002

Graduate School of Mathematical Science

The University of Tokyo, Japan

**Hyper** plane arrangements and  
Japanese  
Australian workshop on  
Real and  
Complex  
Singularities



Graduate School of  
**MATHEMATICAL SCIENCES**  
THE UNIVERSITY OF TOKYO

## Chairpersons (tentative assignment)

	room	morning	13:00 – 15:00	15:00 –
2 (Mon)	002	Gavril Farkas	Stephan Tillmann	Laurentiu Maxim
	128	—	Kentaro Saji	—
3 (Tue)	002	Anthony Henderson	Christin Bibby	Mike Eastwood
	128	—	Susumu Tanabe	—
4 (Wed)	002	Marian Aprodu	—	—
5 (Thu)	002	Claudiu Raicu	Graham Denham	Mario Salvetti
	056	—	Zhou Zhang	—
6 (Fri)	002	Joana Cirici	Ruibin Zhang	Mutsuo Oka

## Organizers

Laurențiu Păunescu (University of Sydney)  
Alex Suciu (Northeastern University)  
Alex Dimca (University of Nice-Sophia Antipolis)  
Toshitake Kohno (University of Tokyo)  
Masahiko Yoshinaga (Hokkaido University)  
Toshizumi Fukui (Saitama Univeisty)

## Supports

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## URL

<http://www.rimath.saitama-u.ac.jp/lab.jp/Fukui/JARCS8/JARCS8.html>

<https://sites.google.com/umich.edu/hyper-jarcs/home>

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**S**ingularities

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## Program

### 2 December (Monday)

- 9:30 opening
- 9:40 – 10:30 **Alex Suciu** (Northeastern University)  
Sigma-invariants, cohomology jump loci, and tropicalization
- 10:40 – 11:30 **Marian Aprodu** (University of Bucharest & IMAR)  
Green's conjecture and vanishing of Koszul modules
- 13:10 – 14:00 **Antoni Rangachev** (University of Chicago)  
Zariski equisingularity and Lipschitz geometry
- Room 128 **Hu He Han** (Northwest A & F University / YNU)  
Constant diameter and constant width for spherical convex bodies
- 14:10 – 15:00 **Uli Walther** (Purdue University)  
Lyubeznik number and Topology
- Room 128 **Shunsuke Ichiki** (Kyushu University)  
Characterization of generic transversality
- 15:20 – 16:10 **Nero Budur** (KU Leuven)  
Jets, quivers, and  $SL_n(\mathbb{Z})$
- 16:20 – 17:10 **Ruibin Zhang** (University of Sydney)  
Equivalence between a category of framed tangles and a category of infinite dimensional representations of quantum  $SL(2)$
- 18:00 – 20:00 Welcome party (Common Room<sup>1</sup>)

### 3 December (Tuesday)

- 9:40 – 10:30 **Toshitake Kohno** (University of Tokyo)  
Higher holonomy and iterated integrals
- 10:40 – 11:30 **Mario Salvetti** (University of Pisa)  
On the  $K(\pi, 1)$ -conjecture for affine Artin groups
- 13:10 – 14:00 **Graham Denham** (University of Western Ontario)  
Singular loci of configuration hypersurfaces
- Room 128 **Kentaro Saji** (Kobe University)  
Contact like relation of vector fields and functions and its applications
- 14:10 – 15:00 **András Lőrincz** (Max Planck Institute)  
Categories of equivariant perverse sheaves
- Room 128 **Shinichi Tajima** (Niigata University)  
An implementation of the Suwa method for computing versal unfoldings of holomorphic foliations
- 15:20 – 16:10 **Zhou Zhang** (University of Sydney)  
Complex Monge-Ampere Equation with Degenerate Cohomology
- 16:20 – 17:10 **Atsufumi Honda** (Yokohama National University)  
Mixed type surfaces in Lorentzian manifolds

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<sup>1</sup>Room 222 on the second floor of the building of Graduate School of Mathematical Science

#### 4 December (Wednesday)

- 9:40 – 10:30 **Gavril Farkas** (Humboldt University Berlin)  
Topological invariants of groups via Koszul modules
- 10:40 – 11:30 **Claudiu Raicu** (University of Notre Dame)  
An effective bound for the Chen ranks conjecture
- 11:40 – 12:00 **Laurentiu Paunescu** (University of Sydney)  
Stefan Papadima – colleague and friend

#### 5 December (Thursday)

- 9:40 – 10:30 **Stephan Tillmann** (University of Sydney)  
A cell-decomposition of decorated moduli space of punctured surfaces
- 10:40 – 11:30 **Dani Matei** (IMAR & University of Tokyo)  
Solvable and nilpotent quotients of 3-manifold and arrangement groups
- 13:10 – 14:00 **Joana Cirici** (University of Barcelona)  
Cohomological operations on complements of subspace arrangements
- Room 056 **Susumu Tanabe** (Galatasaray University)  
Period integrals associated to an affine Delsarte type hypersurface
- 14:10 – 15:00 **Christin Bibby** (University of Michigan)  
The "generating function" of orbit configuration spaces
- 15:20 – 16:10 **Anthony Henderson** (University of Sydney)  
Braid groups of normalizers of reflection subgroups
- 16:20 – 17:10 **Poster session** (Room 056)
- 18:00 – 20:00 Conference Dinner (Restaurant Kanran<sup>2</sup>)

#### 6 December (Friday)

- 9:40 – 10:30 **Florian Pop** (University of Pennsylvania)  
Variants of the Grothendieck-Teichmüller Group
- 10:40 – 11:30 **Laurentiu Maxim** (University of Wisconsin-Madison)  
Defect of Euclidean distance degree
- 13:10 – 14:00 **Mihai Tibar** (University of Lille)  
Concentration of curvature and Lipschitz invariants of holomorphic functions of two variables
- 14:10 – 15:00 **Mike Eastwood** (Adelaide University)  
Some special geometries in dimension five
- 15:20 – 16:10 **Takuro Abe** (Kyushu University)  
Free arrangements, combinatorics and geometry
- 16:20 – 17:10 **Masahiko Yoshinaga** (Hokkaido University)  
Icosidodecahedron and Milnor fiber of hyperplane arrangements

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<sup>2</sup>All speakers from abroad are invited. Participants who want to join is asked to sign on the registration sheet at the entrance of Room 003 till 5PM Tuesday. We are able to accept up to the restaurant capacity (45 people) and the cost is 4000 yen. The restaurant is on the second floor of the Komaba Faculty House inside the campus. (<http://leversonverre-tokyo.com/restaurant/kanran/>)

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## **Abstracts of Talks (November 29, 2019's version)**

**Takuro Abe** (Kyushu University)

### **Free arrangements, combinatorics and geometry**

An arrangement is a finite set of hyperplanes in a vector space. It is said to be free if the logarithmic derivation module, i.e., the polynomial vector fields tangent to hyperplanes in an arrangement is a free module. By Terao's factorization, freeness is closely related to the Poincare polynomial of the complement of an arrangement when the base field is the complex number field. It has been a long standing conjecture whether the freeness depends on the combinatorial data of an arrangement, that is still open. We introduce several new developments on this conjecture, including the division theorem, and combinatorial dependency of the addition-deletion theorems. Also, the recent topics on the relation between the Solomon-Terao algebras and the regular nilpotent Hessenberg varieties are explained.

**Marian Aprodu** (University of Bucharest & IMAR)

### **Green's conjecture and vanishing of Koszul modules**

I report on a joint work with G. Farkas, S. Papadima, C. Raicu and J. Weyman. Koszul modules are multi-linear algebra objects associated to an arbitrary subspace in a second exterior power. They are naturally presented as graded pieces of some Tor spaces over the dual exterior algebra. Koszul modules appear naturally in Geometric Group Theory, in relation with Alexander invariants of groups. We prove an optimal vanishing result for the Koszul modules, and we describe explicitly the locus corresponding to Koszul modules that are not of finite length. We use representation theory to connect the syzygies of rational cuspidal curves to some particular Koszul modules and we prove that our vanishing result is equivalent to the generic Green conjecture.

**Christin Bibby** (University of Michigan)

### **The "generating function" of orbit configuration spaces**

As countless examples show, it can be fruitful to study a sequence of complicated objects all at once via the formalism of generating functions. We apply this point of view to the homology and combinatorics of orbit configuration spaces, using the notion of twisted commutative algebras, which essentially categorify exponential generating functions. With this idea, we will describe a factorization of the orbit configuration space "generating function" into an infinite product, whose terms are surprisingly easy to understand. Beyond the intrinsic aesthetic of this decomposition and its quantitative consequences, it encodes all primary, secondary, and higher representation stability phenomena. This is joint work with Nir Gadish.

**Nero Budur** (KU Leuven)

**Jets, quivers, and  $SL_n(\mathbb{Z})$**

Using jet schemes and quivers, we show that the space of representations of the fundamental group of a compact Riemann surface of genus at least two has rational singularities. We apply this to show that the number of irreducible complex representations of  $SL_n(\mathbb{Z})$  of dimension at most  $m$  grows at most as the square of  $m$ , for a fixed  $n > 2$ .

**Joana Cirici** (University of Barcelona)

**Cohomological operations on complements of subspace arrangements**

Deligne's theory of weights provides a powerful approach to study the cohomology of algebraic varieties. In this talk, I will explain how we use this theory to deduce results for higher cohomological operations (Massey products) on the cohomology of complements of subspace arrangements. We will see that, while mixed Hodge theory implies the vanishing of all Massey products over the rationals, the theory of Galois actions in étale cohomology implies partial vanishing results when working with torsion coefficients. This is joint work with Geoffroy Horel.

**Graham Denham** (University of Western Ontario)

**Singular loci of configuration hypersurfaces**

A finite graph determines a Kirchhoff polynomial, which is a squarefree, homogeneous polynomial in a set of variables indexed by the edges. The Kirchhoff polynomial appears in an integrand in the study of particle interactions in high-energy physics, which provides some incentive to study the motives and periods arising from the projective hypersurface cut out by such a polynomial. From this perspective, work of Bloch, Esnault and Kreimer (2006) suggested that the more natural object of study is, in fact, a polynomial determined by a hyperplane arrangement, which is closely related to the basis generating polynomial of the associated matroid. I will describe joint work with Mathias Schulze and Uli Walther on the singular loci of such polynomials.

**Mike Eastwood** (Adelaide University)

**Some special geometries in dimension five**

We can define a geometry on a smooth manifold as a smoothly varying family of algebraic varieties in its projective tangent bundle. When these varieties are hypersurfaces, this is a "casual structure" in the sense of Makhmali and, when these hypersurfaces are spheres, this is a Lorentzian conformal structure. Especially interesting examples occur in dimension four and five: to be discussed in this talk.

**Gavril Farkas** (Humboldt University Berlin)

**Topological invariants of groups via Koszul modules**

We provide a uniform vanishing result for the graded components of the finite length Koszul module associated to a subspace  $K$  inside the second exterior product of a vector space. This purely algebraic statement has interesting applications to the study of a number of invariants associated to finitely generated groups, such as the Alexander invariants, the Chen ranks, or the degree of growth and nilpotency class. For instance, we explicitly bound these invariants in terms of the first Betti number for the maximal metabelian quotients of the Torelli group or any non-fibred Kaehler group. Joint work with Aprodu, Papadima, Raicu and Weyman.

**Hu He Han** (Northwest A & F University / Yokohama National University)

**Constant diameter and constant width for spherical convex bodies**

In this talk we show that a spherical convex body  $C$  is of constant diameter  $\alpha$  if and only if  $C$  is of constant width  $\alpha$ , for  $0 < \alpha < \pi$ . (This is a joint work with Denghui Wu)

**Anthony Henderson** (The University of Sydney)

**Braid groups of normalizers of reflection subgroups**

Various results are known about the normalizers of reflection subgroups in a complex reflection group. At the most basic level, in certain situations it is known that the normalizer is a semidirect product of the reflection subgroup and a complementary subgroup. Ivan Marin gave a topological definition of a group which should be thought of as the braid group of such a normalizer, and raised the question of whether it has a similar semidirect product decomposition. In joint work with Thomas Gobet and Ivan Marin, we show that this holds in the Coxeter case but not in general. One corollary is a standard basis for the Hecke algebra associated to the normalizer of a standard parabolic subgroup in a finite Coxeter group.

**Atsufumi Honda** (Yokohama National University)

**Mixed type surfaces in Lorentzian manifolds**

A mixed type surface is a connected regular surface in a Lorentzian 3-manifold with non-empty spacelike and timelike point sets. The induced metric of a mixed type surface is a signature-changing metric, and their lightlike points may be regarded as singular points of such metrics. In this talk, we introduce several fundamental invariants along lightlike points, such as the lightlike singular curvature and the lightlike normal curvature. Such invariants can be used to characterize the boundedness of Gaussian curvature at a lightlike points, the Gauss-Bonnet type formula for mixed type surfaces with bounded Gaussian curvature and isometric realization problem for signature-changing metric. This talk is based on arXiv:1811.11392, arXiv:1905.03367, and arXiv:1908.01967.

**Shunsuke Ichiki** (Kyushu University)

**Characterization of generic transversality**

In this talk, the notion of generic transversality and its characterization are given. The characterization is also a further improvement of the basic transversality result and its strengthening which was given by John Mather.

**Toshitake Kohno** (the University of Tokyo)

**Higher holonomy and iterated integrals**

The purpose of this talk is to explain a method to extend monodromy representations of flat connections to higher categories. We develop a method to construct representations of the infinity-category of homotopy infinity-groupoid of a manifold by means of K.-T. Chen's formal homology connections. In particular, we describe 2-holonomy maps for hyperplane arrangements and discuss higher category extensions of KZ connections. This construction provides representations of the 2-category of braid cobordisms. We also describe representations of the fundamental 2-groupoids of the complement of hyperplane arrangement.

**András Lőrincz** (Max Planck Institute)

**Categories of equivariant perverse sheaves**

In this talk, I will discuss some results concerning the category of perverse sheaves on stratified spaces. Under suitable finiteness conditions, such a category is equivalent to the category of finite-dimensional representations of a finite quiver. This is the case when one considers equivariant perverse sheaves on an algebraic variety with finitely many orbits under the action of an algebraic group. We describe such quivers explicitly for some irreducible representations of complex reductive groups and toric varieties. In these cases we use the results to find the explicit D-module structure of local cohomology modules supported in orbit closures.

**Dani Matei** (IMAR & University of Tokyo)

**Solvable and nilpotent quotients of 3-manifold and arrangement groups**

We relate cohomological invariants of finitely presented groups such as cup products, and more generally Massey products, and their corresponding cohomology jumping loci, to representations into triangular and unipotent matrices. We provide applications to fundamental groups of 3-manifolds and to fundamental groups of complements to arrangements.

**Laurentiu Maxim** (University of Wisconsin-Madison)

**Defect of Euclidean distance degree**

Two well studied invariants of a complex projective variety are the unit Euclidean distance degree and the generic Euclidean distance degree. These numbers give a measure of the algebraic complexity for nearest point problems of the algebraic variety. It is well known that the latter is an upper bound for the former. While this bound may be tight, many varieties appearing in optimization, engineering, statistics, and data science, have a significant gap between these two numbers. In this talk, I will explain how to compute the difference of these two invariants (usually referred to as the defect of ED degree) for a smooth complex projective variety, by using classical techniques in Singularity Theory. (Joint work with Jose Rodriguez and Botong Wang.)

**Laurentiu Paunescu** (The University of Sydney)

**Stefan Papadima – colleague and friend**

I will talk about my memories of a very special friend.

**Florian Pop** (University of Pennsylvania)

**Variants of the Grothendieck-Teichmueller Group**

The Grothendieck-Teichmueller Group GT was introduced by Drinfel'd and Ihara, in the attempt to answer the question about giving a combinatorial/topological description of the absolute Galois group  $G_{\mathbb{Q}}$  of the field of rational numbers. Unfortunately, the question whether  $GT = G_{\mathbb{Q}}$  is wide open. I will discuss old and new results about GT, including its “birational variant” which turns out to be equal to  $G_{\mathbb{Q}}$ , thus giving a combinatorial/topological description of  $G_{\mathbb{Q}}$ . Finally, time permitting, I will mention work in progress on a “hyperplane arrangements” variant of GT and its relation with  $G_{\mathbb{Q}}$ .

**Claudiu Raicu** (University of Notre Dame)

**An effective bound for the Chen ranks conjecture**

Proposed by Suciu in the early 00s, the Chen ranks conjecture gives a precise formula



for the Chen ranks of hyperplane arrangement groups. Examples show that this formula may fail in low degrees, but it holds eventually as discussed in work of Cohen and Schenck. I will explain an approach for finding an effective bound, in terms of the first Betti number of the arrangement group, for the degrees where the Chen ranks formula holds. Joint work with Marian Aprodu, Gabi Farkas, and Alex Suciu.

**Antoni Rangachev** (University of Chicago)

**Zariski equisingularity and Lipschitz geometry**

In this talk I will describe an approach to showing that Zariski equisingular families of hypersurfaces are Lipschitz equisingular in all dimensions. It is based on the well-known observation that Zariski equisingular families are birational to families of quasi-ordinary singularities having the same characteristic exponents. A singularity is called quasi-ordinary if there exists a finite map from it to an affine space whose discriminant is contained in a normal crossing divisor. I will show that two germs of quasi-ordinary singularities are biLipschitz equivalent if and only if they have the same normalized characteristic exponents. This is joint work with Hussein Mourtada and Bernard Teissier.

**Kentaro Saji** (Kobe University)

**Contact like relation of vector fields and functions and its applications**

In this talk, we introduce an equivalence relation for pairs of a function and a vector field, such that their relations are the same. After considering classification and recognition, we study applications of the equivalence relation to criteria for singularities and differential geometry of singular points of surfaces.

**Mario Salvetti** (University of Pisa)

**On the  $K(\pi, 1)$ -conjecture for affine Artin groups**

We discuss a recent joint paper with Giovanni Paolini where we gave a proof of the well-known  $K(\pi, 1)$ -conjecture for Artin groups of affine type.

**Alex Suciu** (Northeastern University)

**Sigma-invariants, cohomology jump loci, and tropicalization**

The Bieri–Neumann–Strebel–Renz invariants  $\Sigma^i(X)$  of a connected, finite-type CW-complex  $X$  are the vanishing loci for the Novikov homology of  $X$  in degrees up to  $i$ . The  $\Sigma$ -invariants live in the unit sphere  $S(X) \subset H^1(X, \mathbb{R})$ ; this sphere can be thought of as parametrizing all free abelian covers of  $X$ , while the  $\Sigma$ -invariants keep track of the geometric finiteness properties of those covers. I will describe a connection between the sets  $\Sigma^i(X)$  and the tropicalizations of the cohomology jump loci  $V^i(X) \subset H^1(X, \mathbb{C}^*)$ , and sketch some applications in the realms of complex algebraic geometry and low-dimensional topology.

**Shinichi Tajima** (Niigata University)

**An implementation of the Suwa method for computing versal unfoldings of holomorphic foliations**

In 1980's, T. Suwa investigated singularities of holomorphic foliations and developed a theory of unfoldings. He gave in particular a method for computing versal unfoldings of codimension one local foliations. In this talk, we consider the Suwa method from the point of view of computational complex analysis. We show that, by utilizing Grothendieck local duality on residues, the Suwa method can be realized as an

algorithm that works on computer algebra systems. The key of our approach is the use of local cohomology. This is a joint work with K. Nabeshima (Tokushima Univ).

**Susumu Tanabe** (Galatasaray University)

**Period integrals associated to an affine Delsarte type hypersurface**

We propose a simple method to calculate concretely period integrals associated to an affine non-compact hypersurface for which the number of terms participating in its defining equation is larger than the dimension of the ambient algebraic torus by two (deformed Delsarte hypersurface). A monomial deformation of a Fermat type polynomial belongs to this class. By interpreting the period integrals as solutions to Pochhammer hypergeometric differential equation, we calculate concretely the irreducible monodromy group of period integrals that correspond to the compactification of the affine hypersurface in a complete simplicial toric variety. As an application of the equivalence between oscillating integral for Delsarte polynomial and quantum cohomology of a weighted projective space, we prove a version of Dubrovin's conjecture on its Stokes matrix.

**Mihai Tibar** (University of Lille)

**Concentration of curvature and Lipschitz invariants of holomorphic functions of two variables**

By combining analytic and geometric viewpoints on the concentration of the curvature of the Milnor fibre, we find some new (discrete) Lipschitz invariants which supplement the (continuous) invariants discovered by Henry and Parusinski in 2003. This reports on jointwork with Laurentiu Paunescu.

**Stephan Tillmann** (The University of Sydney)

**A cell-decomposition of decorated moduli space of punctured surfaces**

Fock and Goncharov's  $A$ -coordinates give a parameterisation of the moduli space of doubly-decorated convex projective structures of finite volume on punctured surfaces. We describe a natural cell decomposition of this space that is invariant under the action of the mapping class group, and highlight several nice properties of this decomposition. This generalises a result of Penner concerning decorated Teichmüller space. This is joint work with Robert Haraway, Robert Löwe and Dominic Tate.

**Uli Walther** (Purdue University)

**Lyubeznik number and Topology**

In 1993, G. Lyubeznik introduced for each commutative local ring  $A = R/I$  containing its residue field a set of integer valued invariants by writing it as a quotient of a regular local ring and appealing to (his) results in local cohomology. In particular, for a projective variety  $X$  one can associate these numbers to the local ring of the cone over  $X$  at the vertex. As one would expect, this connects the corresponding Lyubeznik numbers to properties of  $X$ . I will discuss the construction of these numbers (including the basic definitions of local cohomology), survey what is known about them in terms of topology, and discuss the latest developments.

**Masahiko Yoshinaga** (Hokkaido University)

**Icosidodecahedron and Milnor fiber of hyperplane arrangements**

The relationship between combinatorial structure of a hyperplane arrangement and the topology of its Milnor fiber is unclear. In the first part, I will survey Papadima-Suciu's beautiful framework for understanding the monodromy eigenspace of the

first cohomology of the Milnor fiber in terms of Aomoto complex with finite field coefficients. Then we show that the icosidodecahedral arrangement (an arrangement of 16 planes associated with the icosidodecahedron) provides a counterexample to a part of Papadima-Suciu's conjecture. The icosidodecahedral arrangement also provide the first example whose 1st homology of the Milnor fiber has a torsion. This talk is based on arXiv:1902.06256.

**Ruibin Zhang** (The University of Sydney)

**Equivalence between a category of framed tangles and a category of infinite dimensional representations of quantum  $SL(2)$**

We introduce a certain category of framed tangles, and prove an equivalence of it with the full subcategory of quantum  $SL(2)$  representations with objects being the tensor products of infinite dimensional projective Verma modules with powers of the 2-dimensional irreducible module. Both categories are shown to be equivalent to the Temperley-Lieb category of type B, which has been widely studied in representation theory. The results generalise the well-known finite dimensional Schur-Weyl duality between quantum  $SL(2)$  and the Temperley-Lieb algebra, which underlies the quantum group theoretical construction of the Jones polynomial. This talk is based on joint work with Kenji Iohara and Gus Lehrer.

**Zhou Zhang** (The University of Sydney)

**Complex Monge-Ampere Equation with Degenerate Cohomology**

The complex Monge-Ampere equation is a central topic in complex differential geometry, especially after the celebrated Calabi-Yau Theorem. We study the extension when the cohomology class is no longer Kahler, i.e. degenerate. Such scenario appears naturally when searching for canonical metric in the general setting. We soon encounter essential difficulties on regularity of the solution. The Kahler-Ricci flow provides an effective way for construction and also to illustrate the situation.

## **Abstracts of Posters (November 29, 2019's version)**

**Atia Afroz** (Saitama University)

### **A bifurcation model for static solutions of modified Fisher equation**

(Joint work with Toshizumi Fukui) We consider bifurcation problem for static solutions of modified Fisher equation, which is a mathematical model with two alleles. The model is stated as Neumann problem of a bounded region in  $\mathbb{R}^n$ , since there is no flow of gene into the region or out of the region. We introduce a bifurcation model for this problem and discuss how it works when the region is a square.

**Sunil K. Chebolu and Papa A. Sisspkho** (Illinois State University)

### **Zero-sum-free tuples and Hyperplane arrangements**

A vector  $(v_1, v_2, \dots, v_d)$  in  $\mathbb{Z}_n^d$  is said to be a zero-sum-free  $d$ -tuple if there is no non-empty subset of its components whose sum is zero in  $\mathbb{Z}_n$ . We let  $G_n^d$  denote the set of all such tuples and we denote the cardinality of this collection by  $\alpha_n^d$ . Using the natural action of  $\text{Aut}(\mathbb{Z}_n)$  on  $G_n^d$ , we obtain results on the numbers  $\alpha_n^d$  including an exact formula when  $d > n/2$ , divisibility results, and some bounds in the general case. When  $n$  is a prime, we relate this problem to counting points in the complement of a certain Hyperplane arrangement and we show that the characteristic polynomial of the Hyperplane arrangement captures this number for sufficiently large primes and more generally for integers that are relatively prime to some determinants. We also show how the numbers  $\alpha_n^d$  arise naturally in the study of Mathieu subspaces in products of finite fields.

**Mizuki Fukuda** (Tokyo Gakugei University, JSPS PD)

### **Gluck twists along Branched twist spins**

A 2-knot is a smoothly embedded 2-sphere in the 4-sphere and a branched twist spin is a special class of 2-knots. In fact, Plotnick showed that a fibered 2-knot is a branched twist spin if and only if its monodromy is periodic. On the other hand, A Gluck twist is known as a surgery of a 4-manifold along an embedded 2-sphere. The Gluck twist may change the diffeomorphism type of the 4-manifold and the embedded 2-sphere. Pao showed that a manifold obtained from the 4-sphere along a branched twist spin is diffeomorphic to the 4-sphere. In this poster, we show that the 2-knot obtained from a branched twist spin by the Gluck twist is also a branched twist spin.

**Brian Hepler** (University of Wisconsin-Madison)

### **Deformation Formulas for Parameterizable Hypersurfaces**

We investigate one-parameter deformations of functions on affine space which define parameterizable hypersurfaces. With the assumption of isolated polar activity at the origin, we are able to completely express the  $L\hat{e}$  numbers of the special fiber in terms of the  $L\hat{e}$  numbers of the generic fiber and the characteristic polar multiplicities of the comparison, a perverse sheaf naturally associated to any reduced complex analytic

space on which the shifted constant sheaf is perverse. This generalizes the classical formula for the Milnor number of a plane curve in terms of double points as well as Mond's image Milnor number. We also recover results of Gaffney and Bobadilla using this framework. We obtain similar deformation formulas for maps from  $\mathbb{C}^2$  to  $\mathbb{C}^3$ , and provide an ansatz for obtaining deformation formulas for all dimensions within Mather's nice dimensions

**Shuhei Honda** (Saitama University)

### **Singularities of central projections**

We present criteria of singularity types of central projections of several surfaces in  $\mathbb{R}^3$ . They are written as conditions about centers  $y$  and surfaces. We aim to describe the set of center  $y$  which is called "bifurcation diagram" such that the singularity types of the central projection change there. And we consider  $\mathcal{A}$ -versality of central projections to judge whether each singular types is generic.

**Kazumasa Inaba** (Iwate University)

### **Join theorem for mixed hypersurface singularities**

Assume that mixed function maps  $g : \mathbb{C}^n \rightarrow \mathbb{C}$  and  $h : \mathbb{C}^m \rightarrow \mathbb{C}$  admit tubular Milnor fibrations. Let  $X_t$  and  $Y_t$  be the Milnor fibers of  $g$  and  $h$  respectively. We denote the join of  $X_t$  and  $Y_t$  by  $X_t * Y_t$ . In this poster, we show that the existence of a homotopy equivalence  $\alpha : X_t * Y_t \rightarrow Z_t$ , where  $Z_t$  is the Milnor fiber of  $g + h$ .

**Masaharu Ishikawa** (Keio University)

### **Complexity of contact 3-manifolds**

A contact 3-manifold is said to be supported by a flow-spine  $P$  if there is a contact form whose kernel gives that contact structure and whose Reeb flow is a flow of  $P$ . It is proved that there is a surjection from the set of positive flow-spines to the set of contactomorphism classes of contact 3-manifolds. This map allows us to define a complexity of contact 3-manifolds by setting it to be the minimum number of vertices of positive flow-spines that support the contact 3-manifold. For example, the complexity is one if and only if it is the 3-sphere with the standard contact structure. We classify contact 3-manifolds with complexity up to 3. Note that these contact 3-manifolds are all tight. We may find an overtwisted contact 3-manifold with complexity 5. This is a joint work with I. Ishii (Keio), Y. Koda (Hiroshima) and H. Naoe (Chuo).

**Yuta Kambe** (Saitama University)

### **Computation of singular points and homology of the Hilbert scheme via its Bialynicki-Birula decomposition.**

Bialynicki-Birula introduced a cell decomposition of a smooth projective variety  $X$  such that a 1-dimensional algebraic torus acts on  $X$  with finitely many fixed points. Nowadays that decomposition is called Bialynicki-Birula decomposition (BB decomposition for short). Thanks to a recent study of Drinfeld or Jelisiejew and Sienkiewicz, we can define BB decomposition of a singular variety with a 1-dimensional algebraic torus action. In my poster, I will introduce a criterion of singularity on fixed points via BB decomposition. As an example, I will deal with Hilbert schemes that are typical varieties having singular points.

**Wataru Komine** (Saitama University)

### **How model completeness apply to real algebraic geometry?**

Ax-Grothendieck theorem states that polynomial map  $\mathbb{C}^n \rightarrow \mathbb{C}^n$  is injection implies it is surjective. The theorem has a simple proof by using completeness of a theory of algebraically closed field. A key of the proof is that the theorem is represented by first-order sentence. A theory of real closed field (RCF) also has completeness and model completeness. We present whether we can use completeness and model completeness of RCF for given claims in real algebraic geometry.

**Naoki Kitazawa** (Kyushu University)

### **Fold maps on 7-dimensional simply-connected closed manifolds**

From Morse functions and singular points, we can know homology groups and some information on homotopy of the manifolds. It is so-called Morse theory, established already in 1950s. If we consider fold maps, which are higher dimensional versions of Morse functions, and consider suitable classes, then we can know more precise information on the topologies and the differentiable structures of the manifolds. Related studies were started by Thom and Whitney in 1950s and since 1990s, Saeki, Sakuma etc. have developed the studies.

Related to this, we present results on explicit fold maps on 7- dimensional simply-connected manifolds including Milnor's exotic spheres and also meanings in algebraic topology and differential topology of manifolds.

**Hironobu Naoe** (Chuo University)

### **Lefschetz fibrations of divides and shadows**

A'Campo proved that the link of any connected divide is fibered, and it was extended by Ishikawa to the case of divides on orientable surfaces. Such a fibration comes from the Lefschetz fibration that canonically corresponds to a divide. The notion of shadows was introduced by Turaev, and he showed that a shadowed polyhedron determines a 4-manifold uniquely (up to diffeomorphism). We give a method for constructing a shadowed polyhedron from a divide. The 4-manifold reconstructed from a shadowed polyhedron admits a Lefschetz fibration if the polyhedron admits a certain structure, which we call an LF-structure. We will show that the shadowed polyhedron constructed from a divide admits this structure and the Lefschetz fibration of this polyhedron is isomorphic to that of the divide. Furthermore, applying the same technique to certain free divides we show that the links of those free divides are fibered with positive monodromy. This is joint work with Masaharu Ishikawa.

**Phil Tosteson** (University of Chicago)

### **Representation Stability and Braid Milnor Fibers**

The of Milnor fiber,  $F_n$ , associated to the  $n$ th braid arrangements carries an action of the alternating group,  $A_n$ , and a monodromy action by the roots of unity  $\mu_{\binom{n}{2}}$ . We discuss how methods from representation stability can be used to determine the the homology  $H_i(F_n, \mathbb{Z})$  for  $n \gg 0$ , as a representation of these groups.

**Tan Nhat Tran** (Hokkaido University)

### **On $A_1^2$ restrictions of Weyl arrangements**

Let  $\mathcal{A}$  be a Weyl arrangement in an  $\ell$ -dimensional Euclidean space. It is well-known that  $\mathcal{A}$  is a free arrangement, and the exponents of  $\mathcal{A}$  are the same as the exponents of the corresponding Weyl group. It is shown by Orlik-Solomon (1983), using the classification of finite reflection groups, that the characteristic polynomial of every restriction of  $\mathcal{A}$  is fully factored. Orlik-Terao (1993) proved a stronger statement that every restriction is free by a case-by-case study, and Douglass (1999)

later gave a uniform proof for the freeness using the representation theory of Lie groups. Nevertheless, describing theoretically the exponents is still a difficult task. The only known general result due to Orlik-Solomon-Terao (1986), asserts that the exponents of any  $A_1$  restriction, i.e., the restriction of  $\mathcal{A}$  to a hyperplane, are given by  $\{m_1, \dots, m_{\ell-1}\}$ , where  $\exp(\mathcal{A}) = \{m_1, \dots, m_\ell\}$  with  $m_1 \leq \dots \leq m_\ell$ . As a next step after Orlik-Solomon-Terao towards understanding the exponents of the restrictions, we will investigate the  $A_1^2$  restrictions, i.e., the restrictions of  $\mathcal{A}$  to subspaces of type  $A_1^2$ . In this presentation, we give a combinatorial description of the exponents of the  $A_1^2$  restrictions and describe bases for the modules of derivations in terms of the related roots, the classical notion defined by Kostant (1955). This is based on a recent joint work (arXiv:1910.05468) with Takuro Abe (Kyushu) and Hiroaki Terao (Tokyo).

**Takeki Tsuchiya** (Saitama University)

**On properness of polynomial maps**  $F(x, y) = (f(x, y), xy)$

We state a condition such that the polynomial map  $\mathbb{C}^2 \rightarrow \mathbb{C}^2$  defined by  $F(x, y) = (f(x, y), xy)$  is proper. As a corollary, we obtain a condition of  $F(x, y) = (P(x, y), Q(x, y))$ ,  $Q$  a quadratic, being proper.

**Dominik Wrazidlo** (Kyushu University)

**Cobordism of Morse functions, and applications to map germs at boundary points**

Cobordism groups of differentiable maps with prescribed singularities are generally studied by means of stable homotopy theory ( see e.g. the works of Rimányi-Szűcs, Ando, Kalmár, Sadykov, and Szűcs). Historically, the topic was pioneered by René Thom, who applied the Pontrjagin-Thom construction to study embeddings of manifolds into Euclidean spaces up to cobordism.

Cobordism relations for Morse functions are based on proper stable maps into the plane (see the works of Ikegami-Saeki, Kalmár, and Ikegami). There are applications and connections to exotic spheres to high-dimensional TQFT, and to cut and paste invariants of manifolds and SKK-groups. In this work, we use explicit methods of geometric topology to compute the cobordism groups of Morse functions on compact manifolds possibly with boundary. In doing so, we generalize previous work of Ikegami-Saeki, Saeki-Yamamoto, Yamamoto, and Wrazidlo. The underlying stable map germs are fold points and cusps at interior points, and boundary fold points, boundary cusps, and  $B_2$  singularities at boundary points. Our approach also applies to the analogous problem for Morse maps into the circle. As an application, we study topological invariants for generic smooth map germs at boundary points into the plane.

**So Yamagata** (Hokkaido University)

**Combinatorics of the Discriminantal arrangement**

Let us fix  $n$  hyperplanes  $\{H_1^0, \dots, H_n^0\}$  of  $\mathbb{C}^k$  in general position and consider the space  $S$  of parallel translates of  $n$  hyperplanes which is defined as  $n$ -tuples  $(H_1, \dots, H_n)$  such that  $H_i \cap H_i^0 = \emptyset$  or  $H_i = H_i^0$  for all  $i$ . One can identify  $S$  with  $n$ -dimensional affine space  $\mathbb{C}^n$  in such a way that  $(H_1^0, \dots, H_n^0)$  corresponds to the origin. For a fixed general position arrangement consider the closed subset  $D$  of  $S$  which is the collection of parallel translations which fails to be general position. This subset  $D$  can be regarded as a hyperplane of  $S \simeq \mathbb{C}^n$ . The collection of hyperplanes  $D$  is called the Discriminantal arrangement which is introduced by Yu.I.Manin and

V. V. Schechtman in 1986. We show that the description of codimension 2 intersections of hyperplanes of the Discriminantal arrangement can be characterized as a quadric in Grassmannian. This poster is based on the joint work with S. Sawada and S. Settepanella.