Optimal closures for equilateral polygons in any dimension

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In numerical computations involving linkages or closed curves, it's typical to represent the curve by an equilateral polygon. As the computation progresses, accumulated roundoff error can lead to curves which are not quite closed.

This motivates a natural question: is there a natural strategy for reclosing the curve which preserves edgelengths, is continuous in the input polygon (at least, as far as possible), and works in any dimension?

In this talk, we present such an algorithm. Viewing the edges of an equilateral polygon as a point cloud on the sphere, our key idea is a proof that the closest closed polygon to a given open polygon is constructed by recentering the edge cloud at its geometric median and renormalizing.