

The Geometry of Polygon Space: Acute Triangles, Convex Quadrilaterals, Flag Means, and More

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Abstract:

“Three Points are taken at random on an infinite Plane. Find the chance of their being the vertices of an obtuse-angled Triangle.”

This is the text of Lewis Carroll’s Pillow Problem #58, from 1884. This and similar problems (e.g., “what’s the probability that a random quadrilateral is convex?”) sparked intense debate in the 19th century as mathematics was just starting to get to grips with the basics of geometric probability.

Using the Grassmannian model of random polygons developed with Cantarella and Deguchi and based on work of Hausmann and Knutson, I will present the exact probability that a triangle is obtuse and that a polygon is convex, as well as precise statements about extreme and average triangles.

Though this work is focused on planar polygons with few edges, it provides a template for integration and clustering in the physically-relevant setting of polygons in space. This is joint work with Laney Bowden, Jason Cantarella, Andrea Haynes, Tom Needham, Aaron Shukert, and Gavin Stewart.