CURVATURE INTEGRALS OF THE REAL MILNOR FIBRE

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Let $f : \mathbb{R}^{n+1} \to \mathbb{R}$ be a polynomial with an isolated critical point at 0 and let $f_t : \mathbb{R}^{n+1} \to \mathbb{R}$ be a one-parameter deformation of $f$. We continue the study of the differential geometry of the real Milnor fiber $C^0_t = f_t^{-1}(0) \cap B^{n+1}_t$, started by Risler and his students and by the author.

More precisely, we express the following limits:

$$\lim_{t \to 0} \lim_{\varepsilon \to 0} \frac{1}{\varepsilon^i} \int_{C^0_t} \sigma_{n-i} \, dx,$$

where $\sigma_{n-i}$ is the $(n-i)$-th symmetric function of curvature, in terms of the following averages of topological degrees:

$$\int_{G(n+1,k)} \deg_0 (f_H) \, dH,$$

where $G(n+1,k)$ is the Grassman manifold of $k$-dimensional planes through the origin of $\mathbb{R}^{n+1}$.

When 0 is an algebraically isolated critical point, we study the limits:

$$\lim_{t \to 0} \lim_{\varepsilon \to 0} \frac{1}{\varepsilon^i} \int_{C^0_t} h_{n-i} \, dx,$$

where the $h_{n-i}$ are extrinsic curvature functions defined by Langevin and Shifrin. We prove that these limits are finite and that they are bounded in terms of the $\mu^i(f_C)$, i.e the Milnor-Teissier numbers of the complexification of $f$.

Similar results in the complex setting have been obtained by Langevin, Griffiths, Kennedy and Loeser.