HARMONIC ANALYSIS, GEOMETRIC ANALYSIS AND PDE WORKSHOP

SAITAMA UNIVERSITY

6th TENURE-TRACK SYMPOSIUM

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ABSTRACTS

Michael Cowling (University of New South Wales)

Uncertainty Principles

(I) *The classical uncertainty principles of Heisenberg and Hardy*
The first is an L^2 inequality: $\|f\|_2 \leq C\|Xf\|_2\|Df\|_2$, where $Xf(x) = xf(x)$ and $Df(x) = df(x)/dx$. The second is an L^∞ inequality: if $f \leq \gamma$ and $\hat{f} \leq \gamma$, where $\gamma(x) = \exp(-x^2/2)$, then $f = c\,\gamma$. We give proofs and some extensions of these results.

(II) *The Beurling uncertainty principle*
This states that if $\iint |f(x)\,\hat{f}(y)| \exp(|xy|)\,dxdy < \infty$, then $f = 0$. There are now three proofs of this result and generalisations, the first using the Phragmén–Lindelöf three lines theorem, the second using the Bargmann transform and the third using Hadamard factorisation.

(III) *Applications of real interpolation*
It is possible to give real variable proofs of some uncertainty inequalities, at the cost of losing exact constants. We illustrate this and show how uncertainty inequalities may be proved on manifolds from information about the heat equation.

(IV) *Logarithmic inequalities*
There are links between uncertainty inequalities and certain inequalities involving logarithms (such as "log-Sobolev inequalities"), going back to ideas of Hirschmann. We explore some of these and their connections with entropy.

Aya Ishizeki (Saitama University)

A decomposition of the Möbius energy and consequences

We consider the Möbius energy for closed curves in $\mathbb{R}^n$, so-called since it is invariant under Möbius transformations. Since the energy was introduced for finding the canonical configuration of knots, the energy density contains negative powers of the intrinsic and extrinsic distance between any two points on the curve, and this causes significant difficulty with the analysis.

We can decompose the energy into three parts, each of which is Möbius invariant. The first part characterizes the proper domain of the energy; the second one plays the role of canceling the singularity of the density; and the third one gives us information about the minimal value of the energy.

The decomposition gives us easy-to-analyze components. For example, although the first and second variational formulae of the original energy had already been derived in the sense of Cauchy's principal value without our decomposition, we can more easily derive simpler expressions and hence obtain certain applicable estimates for the variational formulae in fractional Sobolev spaces (as well as other spaces) using our decomposition.

Furthermore, the Möbius invariance of each component gives information concerning the minimizers of the energies.

This is a joint work with Prof. Takeyuki Nagasawa (Saitama University).

Sanghyuk Lee (Seoul National University)

Stability of the Brascamp–Lieb inequalities and applications

This talk is concerned about multilinear estimates of which usefulness and importance in harmonic analysis have been proven recently. We show that the bounds for the general Brascamp-Lieb inequality are uniformly finite under small perturbation of the underlying linear transformations. As applications we obtain multilinear Fourier restriction, Kakeya-type estimates, and nonlinear variants of the Brascamp–Lieb inequality. This is a joint work with Jon Bennett, Neal Bez, and Taryn Flock.

Akihiko Miyachi (Tokyo Woman's Christian University)

Bilinear pseudo-differential operators with symbols in the Hörmander class

Hörmander's symbol class $S^m_{\rho,\delta}$ consists of smooth functions $\sigma(x,\xi)$, $(x,\xi) \in \mathbb{R}^n \times \mathbb{R}^n$, that satisfy

$$|\partial_\xi^\alpha \partial_x^\beta \sigma(x,\xi)| \lesssim (1+|\xi|)^{m-\rho|\alpha|+\delta|\beta|}.$$

The L^p boundedness of linear pseudo-differential operators with symbols in the class $S^m_{\rho,\delta}$ is now well-known. Important problems were to prove the L^p boundedness with the critical order m in the case $0 \le \rho < 1$, which were solved by Calderón and Vaillancourt for the case $m = \rho = \delta = 0$ and by C. Fefferman for the case $m = -n(1-\rho)/2$, $0 < \rho < 1$. We consider the similar problem for bilinear pseudo-differential operators and give some partial results. This is a joint work with Naohito Tomita.

Takayoshi Ogawa (Tohoku University)

Global behavior of a drift-diffusion system in higher dimensions

The large time behavior of the solution to the drift-diffusion system is known to be unstable than the case of the lower dimensions. We are going to show a large time behavior of the solution of a drift-diffusion system in higher dimensions regarding its b-th order moment based on the virial laws and modified Shannon's inequality. In particular either the finite time blowing up of the solutions or unboundedness of global solution is shown for a large initial data. We also show the behavior of the blowing up solutions.

Hiroki Saito (Kogakuin University)

General maximal operators and the Reverse Hölder classes

In this talk we discuss a theory of weights for general maximal operators. By a basis we mean a collection of open and bounded sets $\mathfrak{B}$. We show that if the general maximal operator $M_{\mathfrak{B}}$ is bounded on $L^p(\mathbb{R}^n)$ for $p > 1$ and the weight w belongs to the Reverse Hölder $RH_{\infty,\mathfrak{B}}$ class, then the weighted maximal operator $M_{\mathfrak{B},w}$ is bounded on $L^p(\mathbb{R}^n, w)$ for $p > 1$. When the general basis $\mathfrak{B}$ has dyadic structure with some certain properties, we investigate the equivalence between the Muckenhoupt class $A_{\infty,\mathfrak{B}}$ and the Reverse Hölder class $RH_{1,\mathfrak{B}}$. Finally, we introduce a theorem concerning general maximal operator which implies the estimate of the weighted Kakeya maximal operators as a corollary. This is joint work with Professor Hitoshi Tanaka.

Mitsuru Sugimoto (Nagoya University)

Optimal smoothing estimates and trace theorems

Our purpose is to study the optimal constant and extremising initial data for a broad class of smoothing estimates for Schrödinger equations, together with trace theorems as their dual estimates. Rather less was known about these problems except for Simon's work which discussed only a special smoothing estimate, but we will almost completely answer them by a systematic treatment. We will also mention a relation between this problem and Mizohata–Takeuchi Conjecture on the well-posedness of Cauchy problem for Schrödinger equation with drift term.

Kotaro Tsugawa (Nagoya University)

Local well-posedness and parabolic smoothing effect of semilinear fifth order dispersive equations on the torus

We consider the Cauchy problem of semilinear fifth order dispersive equations under the periodic boundary condition. We show the following results. When the nonlinear term is non-parabolic resonance type, we have the local well-posedness on $(-T, T)$. On the other hand, when the nonlinear term is parabolic resonance type, the local well-posedness holds with the parabolic smoothing effect only on either $[0, T)$ or $(-T, 0]$ and a nonexistence result holds on the other time interval.

Po Lam Yung (Chinese University of Hong Kong)

Failure of some critical Sobolev embeddings

I will first give a survey of some recent advances by Bourgain, Brezis, Lanzani, Stein and van Schaftingen, about some compensation phenomena for the failure of certain critical Sobolev embeddings. Then I will discuss various extensions and applications of these results, based on joint works with Yi Wang, Sagun Chanillo and Jean van Schaftingen.

Organisers
Takeyuki Nagasawa (Saitama University)
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Yohei Sato (Saitama University)
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