RIMS Symposium (Open)

Harmonic Analysis and Nonlinear Partial Differential Equations

Date: 24 June – 26 June 2024

Location: Room 420, Research Institute for Mathematical Sciences
Kyoto University, Kyoto 606-8502

Organisers: Neal Bez (Saitama University)
Yutaka Terasawa (Nagoya University)

Program

Monday 24 June

13:15〜14:15 Takahisa Inui (Osaka University)
Global dynamics for threshold even solutions to the nonlinear Schrödinger equation with a repulsive delta potential

14:30〜15:30 Xiaodan Zhou (Okinawa Institute of Science and Technology)
Uniqueness of Green functions in metric spaces, yes and no

15:45〜16:45 Luz Roncal (Basque Center for Applied Mathematics)
Landis-type results for discrete equations

Tuesday 25 June

9:00〜10:00 Ye Zhang (Okinawa Institute of Science and Technology)
Semiconcavity results of squared sub-Riemannian distance on Heisenberg group and applications to PDEs

10:15〜11:15 Sanghyuk Lee (Seoul National University)
Problems related to the Hermite operator I

11:15〜13:15 Lunch

13:15〜14:15 Diogo Oliveira e Silva (Instituto Superior Técnico)
Sharp extension inequalities on finite fields

14:30〜15:30 Anthony Gauvan (Saitama University)
Optimisers for geometric Barthe–Wolff inequality

15:45〜16:45 Sanghyuk Lee (Seoul National University)
Problems related to the Hermite operator Ⅱ

Wednesday 26 June

9:30〜10:30 Mikihiro Fujii (Kyushu University)
Ill-posedness of the stationary and time-periodic solutions to the 2D Navier–Stokes equations on the whole plane

10:45〜11:45 Minsuk Yang (Yonsei University)
Liouville-type theorems for the 3D stationary Navier–Stokes equations

Abstracts

Sanghyuk Lee (Seoul National University)
Problems related to the Hermite operator I, II
Abstract: In this series of talks we are concerned with problems associated with the Hermite operator $\mathcal{H} = -\Delta + |x|^2$, whose eigenfunctions are the Hermite functions. These problems include the sharp L^p bound on spectral projection and the L^p convergence of Bochner–Riesz means. In a local setting, the problems are similar to their counterparts associated with $-\Delta$. However, the problems in the global space exhibit considerable differences that lead to significant difficulties compared with the local problems. I will present recent progress regarding those problems, including applications to strong unique continuation for the heat equation.

Mikihiro Fujii (Kyushu University)
Ill-posedness of the stationary and time-periodic solutions to the 2D Navier–Stokes equations on the whole plane
Abstract: We consider the incompressible Navier–Stokes equations on the whole plane. In contrast to the initial value problem, the solvability of the stationary and time-periodic solutions to the 2D Navier–Stokes equations has long been open. In this talk, we solve this problem negatively. Our method is based on the contradiction argument via the large time behavior of the nonstationary Navier–Stokes flow with the stationary external forces.

Anthony Gauvan (Saitama University)
Optimisers for geometric Barthe–Wolff inequality
Abstract: The Brascamp–Lieb inequality is a far-reaching common generalisation of well-known multilinear functional inequalities on euclidean spaces, such as the Hölder, Loomis–Whitney and Young convolution inequalities. Recently, Barthe and Wolff made a systematic study of "inverse" Brascamp–Lieb inequalities. In this talk, we will describe the optimisers for geometric instances of those inverse inequalities.

Takahisa Inui (Osaka University)
Global dynamics for threshold even solutions to the nonlinear Schrödinger equation with a repulsive delta potential
Abstract: We consider the nonlinear Schrödinger equation with a repulsive delta potential in one-dimensional space. This equation has even ground state solutions, which provide the threshold for even solutions from the viewpoint of global dynamics. The even solutions below the threshold either scatter or blow up. In this study, we investigate the global dynamics of the even solutions at the threshold.

Diogo Oliveira e Silva (Instituto Superior Técnico)
Sharp extension inequalities on finite fields
Abstract: Sharp restriction theory in euclidean space and the extension problem on finite fields have both received much attention in the last two decades, but so far they have not intersected. In this talk, we discuss our first results on sharp restriction theory on finite fields. Even though our methods for dealing with paraboloids and cones borrow some inspiration from their euclidean counterparts, new phenomena arise which are related to the underlying arithmetic and discrete structures. This is based on recent joint work with Cristian González-Riquelme.

Luz Roncal (Basque Center for Applied Mathematics)
Landis-type results for discrete equations
Abstract: The nomenclature "Landis-type results" concern the interest in the maximum vanishing rate of solutions to equations with potentials. We prove Landis-type uniqueness results for both the semidiscrete heat and the stationary discrete Schrödinger equations. The results are obtained through quantitative estimates within a spatial lattice which manifest an interpolation phenomenon between continuum and discrete scales. In the case of the elliptic equation, these quantitative estimates exhibit a rate decay which, in the range close to continuum, coincides with the same exponent as in the classical results of the Landis conjecture in the Euclidean setting. The strategy is inspired by the classical continuum approach, namely logarithmic convexity and Carleman-type estimates. Joint work with Aingeru Fernández-Bertolin and Diana Stan.

Minsuk Yang (Yonsei University)
Liouville-type theorems for the 3D stationary Navier–Stokes equations
Abstract: We deal with a few recent Liouville-type theorems for the 3D stationary Navier–Stokes equations, which were derived using iteration methods.

Ye Zhang (Okinawa Institute of Science and Technology)
Semiconcavity results of squared sub-Riemannian distance on Heisenberg group and applications to PDEs
Abstract: On Euclidean spaces, the squared distance function $|\cdot|^2$ is a convex function as well as a semiconcave function. When it comes to the simplest sub-Riemannian manifold, the Heisenberg group, due to the appearance of the cut locus (the set of points where geodesics cease to be shortest) and the abnormal set (the set of endpoints of abnormal length minimizers), the squared sub-Riemannian distance cannot be locally semiconcave nor locally semiconvex. However, if we consider the corresponding weaker horizontal convexity (or h-convexity in short) on Heisenberg group instead, it turns out that the h-semiconcavity holds for the squared sub-Riemannian distance but the h-semiconvexity still fails. We have two applications to PDEs:

1. the viscosity solution of some non-coercive evolutive Hamilton–Jacobi equations given by the associated Hopf–Lax formula is h-semiconcave but it will break the h-semiconvexity;

2. h-logconcavity is not preserved by the Dirichlet heat flow by a further study of the heat kernel.

Xiaodan Zhou (Okinawa Institute of Science and Technology)
Uniqueness of Green functions in metric spaces, yes and no
Abstract: In Euclidean space $\mathbb{R}^n$, a result of Kichenassamy and Verón shows that the n-Laplace operator

$$\mathcal{L}_n u := \operatorname{div}(|\nabla u|^{n-2}\nabla u)$$

admits a unique global Green function, i.e., there is a unique, properly normalized singular solution which blows up to $+\infty$ at the origin and converges to $-\infty$ at infinity. The argument was later simplified and extended to the Carnot group by Balogh, Holopainen and Tyson, thus establishing uniqueness of global Green functions in the conformal case $p = n$ in this geometry.

The purpose of this talk is to discuss the uniqueness of Q-harmonic Green functions for $Q > 1$ in the setting of complete metric spaces (X, d, μ) equipped with an Ahlfors Q-regular Borel measure μ, and a Poincaré inequality. Over the past 20 years, many aspects of first-order calculus have

been systematically developed in such spaces and spaces satisfying these conditions include Euclidean spaces, Riemannian manifolds, and many nonEuclidean spaces like sub-Riemannian manifolds such as the Heisenberg group, Gromov-Hausdorff limits of manifolds with lower Ricci curvature bounds, visual boundaries of certain hyperbolic buildings, etc. For our particular question about the uniqueness of Green functions, it turns out that the answer depends on the availability of an inner product on the generalized cotangent space of the metric space.

The talk is based on two joint works, one with Mario Bonk and Luca Capogna, and the other with Anders Björn, Jana Björn and Sylvester Eriksson-Bique.