Japanese Australian workshop on Real and Complex Singularities

Dedicated to the memory of Tzee-Char Kuo

28 September – 1 October, 2026

The Quad Seminar Room S418 (McRae)
The Quadrangle, Camperdown Campus, University of Sydney

Abstracts of Talks

Takuro Abe, Rikkyo University
Hyperplane arrangements and graph theory (tentative)

Given a simple graph, we can associate a graphic arrangement corresponding to the edges in the graph, and there is a one to one correspondence between graphs and sugarrangements of a braid arrangement. To an arrangement we can associate the module of logarithmic differential 1-forms, which describes the singularity of the arrangement. In this talk, we give a completely new approach to describe this module by using Stanley-Reisner theory. Explicitly, we can show that the module is the same as the cover ideal of the complementary graphs. We give several applications of Stanley-Reisner theory to algebra of graphic arrangements. In particular, we show that the topology of several complexes coming from a graph determines algebraic invariants including projective dimensions, local cohomologies and Castelnouvo-Mumford regularity.

Ying Chen, HaiNan University
Relative Lipschitz triviality of analytic deformations

In this talk, we investigate the notion of relative bilipschitz equivalence for map germs. Motivated by the relative Kuiper-Kuo condition introduced by Bekka and Koike, we prove that analytic deformations of map germs are relatively bilipschitz trivial under a suitable inequality assumption. Furthermore, for deformations of function germs, we show that the invariance of the integral closure of the Jacobian ideal is sufficient for relative triviality.

Zbigniew Jelonek, Institute of Mathematics, Polish Academy of Science
Bi-Lipschitz equivalent cones with different degrees

We show that for every $k \geq 3$ there exist complex algebraic cones of dimension k with isolated singularities, which are bi-Lipschitz and semi-algebraically equivalent but have different degrees. We also prove that homeomorphic projective hypersurfaces with dimension greater than 2 have the same degree. In the final part of the paper, we classify links of real cones with base $\mathbb{P}^1 \times \mathbb{P}^2$. As an application we give an example of three four-dimensional real algebraic cones in $\mathbb{R}^8$ with isolated singularity which are semi-algebraically and bi-Lipschitz equivalent but have non-homeomorphic bases. We discover also some new tools to study the links of real algebraic varieties. Moreover, we give examples of real algebraic manifolds, which are not diffeomorphic to projective manifolds of odd degree.

Atsuko Katanaga, Iwate University
Topology of the links of cD_n singularities for $n > 4$

We investigate the rank of the second integral homology group of the link of a cD_n singularity for $n > 4$, under the assumption that the singularity is Newton non-degenerate. In the weighted homogeneous case, we further determine the second homology group of the link, including its torsion subgroup, when the singularity is a Thom-Sebastiani sum of singularities of Brieskorn-Pham, cyclic, or chain type. This is a joint work with Masaharu Ishikawa.

Satoshi Koike, Hyogo University of Teacher Education
When and how I knew Tzee-Char Kuo

Through meaningful interactions with Tzee-Char Kuo, I have learned his contributions to Mathematics and his philosophy. In this talk I introduce some of his contributions.

Gus Lehrer, The University of Sydney
The Milnor fibre of an arrangement

I will describe some progress in this difficult problem, based on the use of the NCP (non-crossing partition lattice)

Laurentiu Maxim, University of Wisconsin-Madison, USA
On the topology of fibers of complex polynomial maps

I will survey classical results and present new developments on the topology of both general and atypical fibers of complex polynomial maps, with particular emphasis on the role of singularities at infinity.

Takashi Nishimura, Yokohama National University, Japan
Characterization of envelopes of line families in $\mathbb{R}^2$

In this talk, a recently discovered answer shall be given to the question "Characterize an envelope created by a given straight line family in $\mathbb{R}^2$ in terms of the family". Several related topics shall be explained as well.

Mutsuo Oka, Tokyo Institute of Technology, Japan
Stability of certain μ-constant families

We consider a μ-constant family $f_t = h_t + R_t$ where the principal part h_t is weighted homogeneous polynomial of degree d under the weight P, $\deg_P R_t > d$ and $h_t = 0$ has an isolated singularity at the origin for $t \neq 0$. We will show that $h_0 = 0$ has also an isolated singularity for 3 variables case.

Mark Perrin, The University of Sydney
A discrete derivative for quasi-bilipschitz equivalences

We discuss the notion of quasi-bilipschitz equivalences as a generalisation of bilipschitz mappings, and discuss the behaviour of tangent-sets under such mappings. We construct a quasi-extension formula for a general quasi-bilipschitz map, which turns out to be bilipschitz, which allows us to extend some of the recent bilipschitz theory to this broader setting.

Laurentiu Paunescu, The University of Sydney
TBA

Milena Radnovic, The University of Sydney
Poncelet porism in singular cases

The celebrated Poncelet porism states that, if there is a closed polygon inscribed in one of the two given conics and circumscribed about the other one, then there are infinitely many such polygons. Usually, it is assumed that the conics are smooth and in a general position, meaning that they have only transversal intersections. In this talk, I will discuss Poncelet porism in the real plane - affine or projective, when that is not the case, i.e. the conics have at least one point of tangency or at least one of the conics is not smooth.

Osamu Saeki, Kyushu University
Murasugi numbers of algebraic knots

We study Murasugi numbers $g(K)$ of $(2n-1)$–dimensional (not necessarily

spherical) knots K in $S^{2n+1} = \partial D^{2n+2}$ for $n \geq 2$, where $g(K)$ is the minimal middle Betti number among those of properly embedded compact oriented $2n$–dimensional submanifolds of D^{2n+2} homotopy equivalent to a bouquet of n–spheres bounded by K. We show that if $n \geq 4$ is even, then for the algebraic knot K_f associated with an isolated critical point of a holomorphic function germ $f : (\mathbf{C}^{n+1}, \mathbf{0}) \to (\mathbf{C}, 0)$, we have $\mu(K_f) = g(K_f)$ if and only if the singularity of f is simple or almost-simple, provided that $H_{n-1}(K_f; \mathbf{Z})$ is torsion free, generalizing a result due to Michel for the case of vanishing $H_{n-1}(K_f; \mathbf{Z})$, where $\mu(K_f)$, the Milnor number, is the middle Betti number of the Milnor fiber for f. This is a joint work with Vincent Blanlœil (University of Strasbourg).

Edson Sampaio, Universidade Federal do Ceará, Brazil
Proof of the metric Arnold corank problem

In this talk, we will approach the Arnold corank problem, posed by Arnold in 1975, which asks whether the corank of holomorphic functions is an ambient topological invariant. We will present a complete positive answer to the metric Arnold corank problem, which asks whether the corank of holomorphic functions is an ambient bi-Lipschitz invariant. Consequently, we will show that for complex hypersurfaces, the multiplicity equal to two is an ambient bi-Lipschitz invariant.

Kiyoshi Takeuchi, Tohoku University
A Ginsburg type theorem for real constructible sheaves and its applications

Ginsburg's theorem describes the characteristic cycles of complex nearby cycle sheaves. In this talk, we present its analogue for real constructible sheaves (adapted to subanalytic stratifications) and give some applications to singularity theory. This is a joint work with Ren Fernandes and Kazuki Kudomi.

Masato Tanabe, RIKEN Center for Interdisciplinary Theoretical and Mathematical Sciences (iTHEMS)
Thom polynomials relative to prescribed maps around the boundary

In one line of applications of Thom polynomial theory, various invariants of immersions have been expressed in terms of singularities of their extension maps (a.k.a. singular Seifert surfaces). In this talk, we aim to place them in

a unified framework, i.e., to establish the foundation of a relative version of Thom polynomial theory. First, we introduce Thom polynomials relative to prescribed maps around the boundary. Second, we show a structure theorem for Thom polynomials relative to framed immersions. Third, we view earlier works from our framework and also give several applications. We are based on Steenrod's obstruction theory and Kervaire's relative characteristic classes. The $\mathcal{K}$-invariance of singularity types plays an important role.

Takahiro Yamamoto, Tokyo Gakugei University
Apparent contours of PL maps of closed surfaces into the plane

In this talk, we define the singular points of PL maps from closed surfaces into the plane from the viewpoint of singular fibers. We then discuss the apparent contours. Furthermore, we introduce the notion of the simple apparent contour of a closed surface associated with a PL map and determine the simple apparent contours for several classes of closed surfaces.

Masahiko Yoshinaga, The University of Osaka
Varchenko-Gelfand algebras of hyperplane arrangements

Shintaro Yoshizawa, Nagoya Mathematical and Information Science Research

The Geometry of Monarch Matrix Approximation: Quiver Representations, Morse Theory, and the Relative-Position Combinatorics of Block Permutations

Extending Helmke–Shayman's Morse theory(1995) to *Monarch matrices* $M = PLPR$, we show the permutation P carries two invariants relative to the block algebra B. The centralizer $B \cap C(P)$, an endomorphism algebra of a quiver representation, governs $\ker D\Phi$, $\dim M_P$, and a full second-order theory (Hessian, Morse index, focal set), and is minimized exactly when P is a single aperiodic n-cycle (the Monarch–Schur theorem). The relative-position algebra $B \cap PBP^\top$ gives a Bruhat-type codimension formula via a generalized Birkhoff polytope, and we prove it always strictly exceeds the first invariant, explaining why the Helmke–Shayman coincidence is special to full flags. Together they yield a three-tier hierarchy governing Monarch approximation geometry. We also show the classical coercivity–Ł ojasiewicz convergence fails for the gradient flow, due to unbounded flat directions from non-closedness of the Monarch image; a quartic Tikhonov penalty restores global convergence to minimal-complexity solutions, giving a geometric ac-

count of weight decay on structured factorized layers.

Ruibin Zhang, The University of Sydney
Categories of polar diagrams

Chong Zheng, Shibaura Institute of Technology/ Waseda University
A Combinatorial Cerf Theory for Discrete Morse Functions

Cerf theory studies generic one-parameter families of smooth functions and describes how Morse data change when the family crosses birth–death singularities. In this talk, we will present a discrete analogue of this picture for Forman's discrete Morse functions on finite simplicial complexes.

The basic idea is to replace a smooth one-parameter deformation by a finite sequence of elementary combinatorial transitions. Among these transitions, the birth–death move creates or cancels exactly one pair of critical cells, giving a discrete counterpart of the birth–death bifurcation in classical Cerf theory. The main theorem states that any two discrete Morse functions on a fixed finite simplicial complex can be connected by a finite sequence of such birth–death transitions.

This gives a Cerf-theoretic structure to the space of discrete Morse data. In particular, by tracing how the underlying complex and the Morse data change under these elementary transitions, one recovers the fundamental theorems of Forman's discrete Morse theory.

Abstracts of Posters

Toshizumi Fukui, Saitama University
Curvature criteria of $\mathcal{A}$-simple singularities $\mathbb{R}, 0 \longrightarrow \mathbb{R}^2, 0$ and their parallel curves

We introduce the notion of curvature parameters for singular plane curves with finite multiplicities and define the notion of curvatures for them. We then provide criteria to determine their singularity types for $\mathcal{A}$-simple singularities. As an application, we investigate singularity types of their parallel curves. This is a joint work with Saiki Hoshino.

Koki Iwakura, Kyushu University
TBA

Kosuke Nishijima, Waseda University
An extension of fractal Euler number via persistent homology

In the context of geometric measure theory, Llorente-Winter introduced the (average) fractal Euler number as a notion of the Euler characteristic for fractals embedded in Euclidean space. However, the class of fractals to which it is applicable remains very limited. In the present paper, we modify this notion by applying perspectives of persistent homology and partly the theory of magnitude, which have recently come from applied topology and category theory. We then demonstrate concrete calculation of our average ph-fractal Euler number for some classically well-known fractals, especially the Cantor dust and Menger sponge which are excluded from Llorente-Winter's approach.