JAPANESE AUSTRALIAN WORKSHOP ON REAL AND COMPLEX SINGULARITIES
4 — 7 NOVEMBER, 2024, SAITAMA UNIVERSITY

4 November, Room 101, Sogo-Kenkyu tou 1 (1st floor of Building 21 on **the campus map**)
5 – 7 November, Room 701, Kenkyu-Kikou tou (7th floor of Building 3 on **the campus map**)

Program

4 November, Monday

9:40 — 10:40 **Mark Perrin**, University of Sydney
Semialgebraic Geometry — Modifying Thom's Lemma

10:50 — 11:50 **Yang Zhang**, University of Queensland
Einstein metrics on homogeneous superspaces

13:30 — 14:30 **Masaaki Umehara**, Institute of Science Tokyo
Unextendability of real analytic map images and its applications

14:40 — 15:40 **Ruibin Zhang**, University of Sydney
Noncommtative spaces for Green's generalised quantisation

15:50 — 16:50 Poster session

18:00 — Welcome Party at **Bar Melin Forest (バル・メリンの森)** in Saitama University

5 November, Tuesday

9:40 — 10:40 **Adam Harris**, University of New England
Quantal Response Equilibria in Game Theory — an application of elementary Catastrophe Theory

10:50 — 11:50 **Kenta Hayano**, Keio University
Characterization of generic parameter families of constraint mappings in optimization

13:30 — 14:30 **Shintaro Akamine**, Nihon University
Singularities of generalized timelike minimal surfaces in Lorentz-Minkowski 3-space

14:40 — 15:40 **Yutaka Matsui**, Kindai University
Introduction to bicomplex hyperfunctions

15:50 — 16:50 **Kiyoshi Takeuchi**, Tohoku University
Cohomological predictions and geometries behind them

6 November, Wednesday

9:40 — 10:40 **Shinichi Tajima**, Niigata University
Computing Camacho-Sad-Suwa indices of logarithmic vector fields along singular curves via Grothendieck local residues

10:50 — 11:50 **Atsufumi Honda**, Yokohama National University
Convexity of space-like projections of submanifolds with co-dimension 2 in Lorentz-Minkowski space

13:30 — 14:30 **Kentaro Saji**, Kobe University
Normal forms of D_4 singularities of fronts and their applications

14:40 — 15:40 **Masato Tanabe**, Hokkaido University
Unstability problem of real analytic maps

15:50 — 16:50 **Masaharu Ishikawa**, Keio Universuty
Atypical values at infinity of real polynomial functions of three variables

7 November, Thursday

9:40 — 10:40 **Mutsuo Oka**, Tokyo University of Science
Almost non-degenerate functions and some applications

10:50 — 11:50 **Christophe Eyral**, Polish Academy of Sciences
On Milnor-Orlik's theorem and simultaneous good resolutions for weighted homogeneous surface singularities

13:30 — 14:30 **Adam Parusiński**, University of Côte d'Azur
Motivic, logarithmic, and topological Milnor fibrations

14:40 — 15:40 **Ayako Kubota**, Saitama University
Invariant Hilbert scheme and resolution of singularities of quotient varieties

15:50 — 16:50 **Laurenţiu Păunescu**, University of Sydney
The Lipschitz type of the geometric directional bundle

18:00 — Conference Dinner at **Monsoon Cafe Saitama-Shintoshin**

A **Restaurents map** nearby / **Application** to Welcome Party and Conference Dinnar

Organizers: Laurentiu Paunescu, Satoshi Koike, Osamu Saeki, Toru Ohmoto and Toshi Fukui

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Japanese
Australian workshop on
Real and
Complex
Singularities
4 — 7 November, 2024, Saitama University

Abstracts

Shintaro Akamine, Nihon University

Singularities of generalized timelike minimal surfaces in Lorentz-Minkowski 3-space

A timelike minimal surface in Lorentz-Minkowski 3-space is a surface whose induced metric is Lorentzian and with vanishing mean curvature. In this talk, we show that various diffeomorphism types of singularities that do not appear on these Riemannian counter parts, such as branched minimal surfaces in Euclidean 3-space and maximal surfaces in Lorentz-Minkowski 3-space, appear on timelike minimal surfaces. This talk is based on the preprint arXiv:2408.00313.

Christophe Eyral, Polish Academy of Sciences

On Milnor-Orlik's theorem and simultaneous good resolutions for weighted homogeneous surface singularities

Let f be a (possibly Newton degenerate) weighted homogeneous polynomial defining an isolated surface singularity at the origin of $\mathbb{C}^3$, and let $\{f_s\}$ be a generic deformation of its coefficients such that f_s is Newton non-degenerate for $s \neq 0$. In this talk, we will show that there exists a *simultaneous good resolution of the family of functions* f_s for all small s, including $s = 0$ which corresponds to the (possibly Newton degenerate) function f. As an application, we will give a new geometrical proof of a weak version of the Milnor–Orlik theorem that asserts that the monodromy zeta-function of f (and hence its Milnor number) is completely determined by its weight, its weighted degree and its Newton boundary. This is a joint work with Mutsuo Oka.

Adam Harris, University of New England

Quantal Response Equilibria in Game Theory — an application of elementary Catastrophe Theory

I will discuss the definition of Quantal Response Equilibrium (QRE), as a generalization of Von-Neumann-Nash Equilibrium, in the context of elementary Games, such as Prisoners' Dilemma. QRE involves a parametric version of game theory, in which equilibrium strategies may be characterized as degenerate critical points of a potential function. This leads to a topological classification of Catastrophe loci in terms of the well-known theory of Thom and Whitney and many others. I will indicate how these mathematical ideas arise, and aim to explain how they come to be part of a revived interest in the role of topology and analytic geometry in Economic Game Theory. This is joint work with Michael Harre and Scott McCallum of Sydney University.

Kenta Hayano, Keio University

Characterization of generic parameter families of constraint mappings in optimization

Constrained optimization is a problem of minimizing objective function(s) within the feasible set that is described by the system of equalities and inequalities of constraint functions. This problem appears in a wide range of academic and industrial tasks, including planning, scheduling, design, development, and operation. The purpose of our study is to understand generic behavior of constraint functions in optimization problems relying on singularity theory of smooth mappings. To this end, we will focus on the subgroup $\mathcal{K}[G]$ of the Mather's group $\mathcal{K}$, whose action to constraint map-germs preserves the corresponding feasible set-germs. In this talk, we will explain the classification of map-germs with small stratum $\mathcal{K}[G]_e$-codimensions, and calculations of the codimensions of the $\mathcal{K}[G]$-orbits of jets represented by germs in the classification lists, and those of the complements of these orbits. If time permits, we will also explain generic behavier of constraint mappings. This is a joint work with Naoki Hamada (KLab Inc.) and Hiroshi Teramoto (Kansai University).

Atsufumi Honda, Yokohama National University

Convexity of space-like projections of submanifolds with co-dimension 2 in Lorentz-Minkowski space

In this talk, we introduce the notion of "essential cocausalily" for submanifolds of co-dimension 2 in Lorentz-Minkowski space. We show that a submanifold of co-dimension 2 in Lorentz-Minkowski space is essentially cocausal if and only if any space-like projection of the submanifold is locally strictly convex. Its applications are also given. This talk is based on a joint work with Toshizumi Fukui (Saitama University) and Masaaki Umehara (Institute of Science Tokyo).

Masaharu Ishikawa, Keio Universuty

Atypical values at infinity of real polynomial functions of three variables

A value of a polynomial map f is said to be *atypical at infinity* if there is no open neighborhood of the value such that f restricted to the preimage of this neighborhood is locally trivial. The set of such values and singular values of f is called the *bifurcation set* of f. There are several studies about this set in both of real and complex cases. However, determining the bifurcation set is still a difficult problem if f has more than two variables. In this talk, we focus on real polynomial functions of three variables and give a characterization of atypical values at infinity by a certain sum of indices of the gradient vector field of f restricted to a sphere with a sufficiently large radius. This work is a 3-dimensional analogy of a result of Coste and de la Puente for real polynomial functions of two variables. This is a joint work with Tat-Thang Nguyen (IMVAST).

Ayako Kubota, Saitama University

Invariant Hilbert scheme and resolution of singularities of quotient varieties

The invariant Hilbert scheme is a moduli space of affine schemes that are stable under an action of a reductive algebraic group. Under a suitable choice of the parameter, there exists a natural morphism, called the quotient-scheme map, from the invariant Hilbert scheme to an affine quotient variety, and it makes the invariant Hilbert scheme a candidate for a resolution of singularities of the quotient variety. In this talk, we look at some examples where the quotient-scheme map gives a resolution of singularities of the quotient variety.

Yutaka Matsui, Kindai University

Introduction to bicomplex hyperfunctions

In this talk, we study bicomplex hyperfunctions introduced by Colombo et al. ([1]) as a natural generalization of classical complex hyperfunctions introduced by Sato ([2]).
Let $V \subset \mathbb{R}^n$ be an open set and $\Omega \subset \mathbb{BC}^n$ a bicomplex neighborhood of V satisfying $V = \mathbb{R}^n \cap \Omega$. Let $\mathcal{O}_{\mathbb{BC}^n}$ denote the sheaf of bicomplex holomorphic functions on $\mathbb{BC}^n$. Colombo et al. proved that the relative cohomology group $H^p_V(\Omega; \mathcal{O}_{\mathbb{BC}^n})$ of $\mathcal{O}_{\mathbb{BC}^n}$ supported in V vanishes if $p \neq 3n$ and defined a bicomplex hyperfunction as an element of the relative cohomology group

$$\mathcal{B}_{\mathbb{BC}^n}(V) = H^{3n}_V(\Omega; \mathcal{O}_{\mathbb{BC}^n}).$$

In this talk, by a functorial method, we reprove the vanishing theorem and prove the idempotent representation theorem of bicomplex hyperfunctions.

[1] F. Colombo, I. Sabadini, D. C. Struppa, A. Vajiac, M. Vajiac, Bicomplex hyperfunctions, Ann. Mat. Pura Appl. 190 (2011), 247–261.

[2] M. Sato, Theory of hyperfunctions I, II, J. Fac. Sci. Univ. Tokyo 8 (1959), 139–193.

Mutsuo Oka, Tokyo University of Science

Almost non-degenerate functions and some applications

We introduce a class of almost Newton non-degenerate functions which is wider than Newton non-degenerate functions. Then we describe their Milnor fibration. This can produce some applications on "Zariski pairs of links" and also "Comments on Briaçon -Speder polynomials." We will explain these applications.

Adam Parusiński, University of Côte d'Azur

Motivic, logarithmic, and topological Milnor fibrations

We compare the topological Milnor fibration and the motivic Milnor fibre of a regular complex function. We give two equivalent constructions: the first one extending the classical Kato-Nakayama log-space, and the second one, a version of the real oriented deformation to the normal cone. We recover A'Campo's model of the topological Milnor fibration, by quotienting the motivic Milnor fibration with suitable powers of $R > 0$, and show that it determines the motivic Milnor fibre.

We also give precise formulas expressing how the motivic and topological Milnor fibrations change under blowings-up. As an application we show that the motivic Milnor fibre is well defined as an element of a suitable Grothendieck ring without requiring the Lefschetz motive to be invertible.

Based on a joint work with J.-B. Campesato and G. Fichou.

Laurenţiu Păunescu, University of Sydney

The Lipschitz type of the geometric directional bundle(joint with Satoshi Koike)

We investigate the behaviour of the geometric directional bundles, associated to arbitrary subsets in $\mathbb{R}^n$, under bi-Lipschitz homeo-morphisms, and give conditions under which their bi-Lipschitz type is preserved. The most general sets we consider satisfy the sequence selection property (SSP) and, consequently, we investigate the behaviour of such sets under bi-Lipschitz homeomorphisms as well. In particular, we show that the bi-Lipschitz images of a subanalytic sets generically satisfy the (SSP) property.

Mark Perrin, University of Sydney

Semialgebraic Geometry — Modifying Thom's Lemma

One of the major tools used in the study of semialgebraic geometry is the cylindrical algebraic decomposition (c.a.d.), that is a partition of $\mathbb{R}^n$ into 'cells'. Given a finite family of polynomials in n variables, a c.a.d. of $\mathbb{R}^n$ can be constructed algorithmically such that each polynomial is sign-invariant on the cells of the decomposition, and thus any semialgebraic subset can be expressed as a union of cells, on each of which the underlying family of polynomials have constant sign. The process of constructing such a c.a.d. starts with a sequence of n projections, followed by a sequence of n liftings.

Thom's Lemma (and its generalisation to several variables) provides a condition that guarantees that we can recover the topology of the original set and, moreover, that the closure of each cell C is a union of cells whose description is obtained in a simple way from the sign conditions that describe C itself. To apply Thom's Lemma, one must ' complete the family ' of polynomials by taking all nonzero derivatives of the members of the polynomials in the family. We explore a modification of the conditions to Thom's Lemma which aims to reduce the size of the resulting family of polynomials, and we provide an algorithmic implementation of it in c.a.d. construction.

Kentaro Saji, Kobe University

Normal forms of D_4 singularities of fronts and their applications

We give $SO(3)$-normal forms of D_4 singularities of fronts. Using these forms, we study geometric properties of D_4 singularities. D_4 singularities of fronts are appeared on parallel surfaces of regular surface at umbilic points and minimal surfaces, a better understanding of these singularities leads to that of these surfaces.

Shinichi Tajima, Niigata University

Computing Camacho-Sad-Suwa indices of logarithmic vector fields along singular curves via Grothendieck local residues

In 1982, C. Camacho and P. Sad defined the index of a holomorphic vector field relative to an invariant non-singular curve and obtained an index formula to prove the existence of an separatrix. Then, A. Lins Neto and T. Suwa generalized the definition of indices to the case where separatirices may have singularities by utilizing a result on logarithmic vector fields due to K. Saito. These results have been extensively utilized by several authors. We consider in this talk Camacho-Sad-Suwa indices in the context of computational complex analysis. We present an effective algorithm for computing Camacho-Sad-Suwa indices of logarithmic vector fields along singular curves. The key of our approach is the concept of Grothendiek local residues.

This is a joint work with T. Shibuta (Kyushu Sangyo Univ.) and K. Nabeshima (Tokyo Univ. of Science).

Kiyoshi Takeuchi, Tohoku University

Cohomological predictions and geometries behind them

The theory of nearby and vanishing cycles for perverse sheaves is useful to obtain various new results on the monodromies of polynomial and rational functions. In this talk, we recall some previous results and discuss their geometric backgrounds. We focus our attention on polynomial functions over affine varieties and rational functions.

Masato Tanabe, Hokkaido University

Unstability problem of real analytic maps

I would like to talk about the real analytic version of the structual stability problem. We show that the infinitesimal C^ω stability does not imply the C^ω stability. To be precise, a proper C^ω map which has a Whitney umbrella or a Swallowtail singularity is C^ω unstable. This is a joint work with Karim Bekka, Satoshi Koike, Toru Ohmoto and Masahiro Shiota.

Masaaki Umehara, Institute of Science Tokyo

Unextendability of real analytic map images and its applications

In the speaker's previous work with Fujimori, Kawakami, Kokubu, Rossman, Yamada and Yang, it was shown that some families of catenoids as space-like constant mean curvature one surfaces in de Sitter 3-space have non-trivial analytic extensions, which admit only cone-like singular points. These analytic extensions are "analytically complete", which implies that they have no further analytic extensions. However, the criterion for analytic completeness there was insufficient for application to the image of analytic maps with much more general singularities.

The goal of the present work is to remedy this, and establish results with broader applicability without the limitation of the dimension of the ambient space, applying subanalytic geometry in singularity theory. More precisely, we give several new criteria for analytic completeness, which will work as useful tools for evaluating unextendablity of analytic maps with generic singularities defined on an m-manifold to a real analytic n-manifold such that $n \geq m$. One type of the criteria is for analytic maps admitting only generic singular points, and the other type of the criteria does not require the identification of singularities.

As an application, we examine the analytic completeness of other remaining families of space-like constant mean curvature one catenoids in de Sitter 3-space, and determine analytic extensions of them.

Ruibin Zhang, University of Sydney

Noncommtative spaces for Green's generalised quantisation

We study a class of noncommutative spaces which provide a natural setting for parafermions in physics, in the same way as supermanifolds for fermions. Such noncommutative spaces are manifolds equipped with sheaves of non-commutative algebras referred to as para-algebras. For any given supermanifold X, there is a family of noncommutative spaces $X(p)$ for the positive integers p, such that $X(1)$ recovers the supermanifold itself. A differential analysis on such noncommutative spaces is developed, which can be readily applied to model physical problems. The affine and projective para-spaces are treated in some detail.

Yang Zhang, University of Queensland

Einstein metrics on homogeneous superspaces

In this talk, we will show how to generalise classical geometric concepts, such as Riemannian metrics and curvatures, to the setting of supermanifolds. We begin by obtaining explicit formulas for the Ricci and scalar curvatures of homogeneous metrics on these spaces. Using these formulas, we solve the Einstein equation on a large class of homogeneous superspaces. This leads to examples of supermanifolds on which there are no Einstein metrics, discrete families of Einstein metrics, and continuous families of Einstein metrics. We also find examples of Ricci-flat metrics, the existence of which indicates that the generalisation of the Bochner theorem to supermanifolds does not hold. This is joint work with Mark Gould, Artem Pulemotov, and Jorgen Rasmussen.